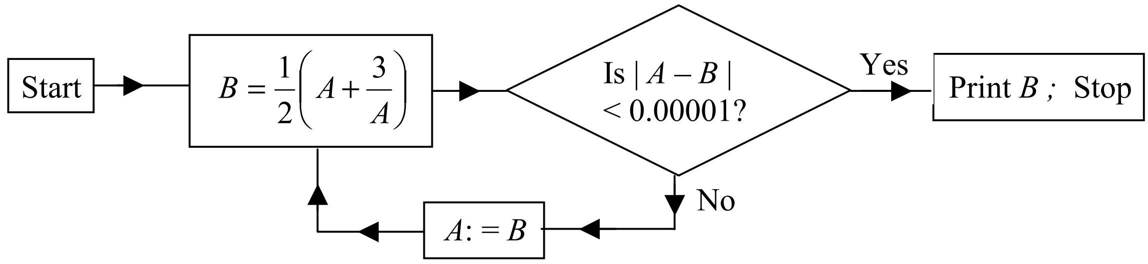
DECISION MATHEMATICS (C) UNIT 1 TEST PAPER 5

1 (i) Prove that in any graph, the sum of the degrees of all the nodes must be an even number. [2]

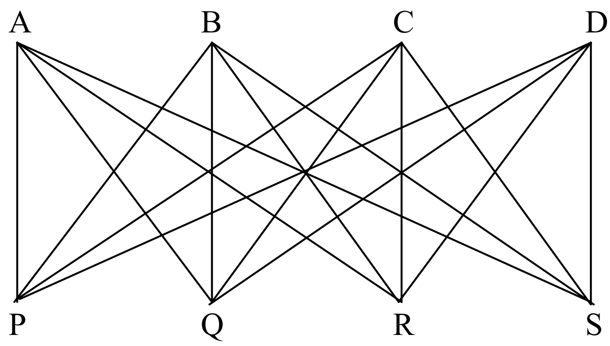
(ii) Hence explain why it is impossible to have an odd number of odd nodes in a graph. [2]

2. (i) Implement the algorithm shown in this flow chart, starting with $A = 2$. List all the intermediate values of A and B. [5]



(ii) What is the algorithm designed to find? [2]

3. In a certain game, the ball must be passed alternately from a player in one team to a player in another team. The two teams, of 4 players each, can be modelled as a bipartite graph $K_{4,4}$:



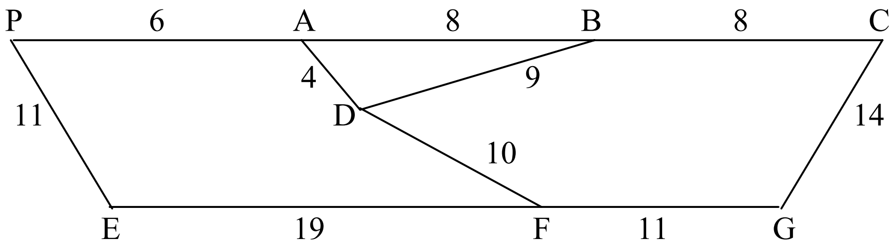
(i) Find how many possible routes there are for the ball, starting and finishing at A, and never going to the same player twice. [2]

(ii) How many distinct routes would there be if there were 10 players in each team? [2]

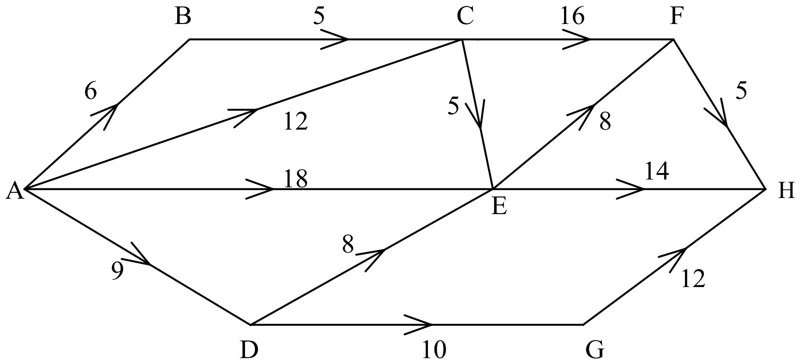
(iii) Traditionally, at the end of the game, each player links hands with two of the opposing team, of whom one is the person opposite. Draw an example to show that the resulting graph may be planar. [3]

4. (i) Briefly describe the algorithm for solving the Route Inspection problem. [2]

(ii) Use this algorithm to find a route of minimum length, which passes at least once along each arc of this network, starting and finishing at P. Give an example of such a route, and state its length. [6]



5. Use Dijkstra's algorithm to find the shortest route from A to H in the network shown. Clearly indicate the order in which you labelled the vertices, and how you determined the shortest route from the labelling.



[9]

6. In the solution of a linear programming problem, it is required to maximise $P = 4x + 5y$, subject to the constraints $4x + y \leq 7$, $5x + 2y \leq 12$ and $y \geq 3x$, (together with $x, y \geq 0$).

   (i) Illustrate these inequalities graphically, and identify the feasible region. [6]

   (ii) Find the values of x and y which produce the maximum value of P, and write down this maximum value of P. [6]

7. A linear programming problem leads to the following equations:
   the function $P = 2x + y + 3z$ must be maximised,
   subject to the constraints $x + y + 2z \leq 6$, $2x + 5y + z \leq 10$; $x \geq 0$, $y \geq 0$, $z \geq 0$.

   (i) Set up the initial Simplex Tableau, and carry out two iterations, starting by using the x-column. [9]

   (ii) State the maximum value of P, and the corresponding values of x, y and z. [2]

   (iii) Explain why your solution is optimal. [2]

DECISION MATHS 1 (C) PAPER 5 : ANSWERS AND MARK SCHEME

1. (i) Each arc has two ends, so total number of ends = $2A$, an even number.
But this is also the sum of the degrees of all the nodes, so it is even — M1 A1

(ii) All even nodes contribute an even amount to the even total; therefore, odd nodes must also contribute an even amount. But an odd number of odd nodes would give an odd contribution, so there must be an even number of odd nodes — M1 A1 — 4

2. (i)

A	2	1.75	1.7321428	1.7320508	M1 A1	
B	1.75	1.7321428	1.7320508	1.7320508	M1 A1	

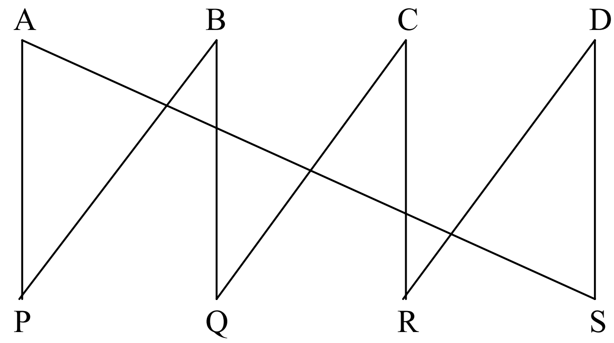
Print out 1.7320508 — A1

(ii) The square root of 3 — B2 — 7

3. (i) From A, there are four choices, then 3, then 3, then 2…, giving — M1
4 x 3 x 3 x 2 x 2 x 1 x 1 x 1 = 144 — A1

(ii) 10 x 9 x 9 x 8 x 8 x 7 x 7 …… x 1 = 1316818944000 — M1 A1

(iii) e.g.

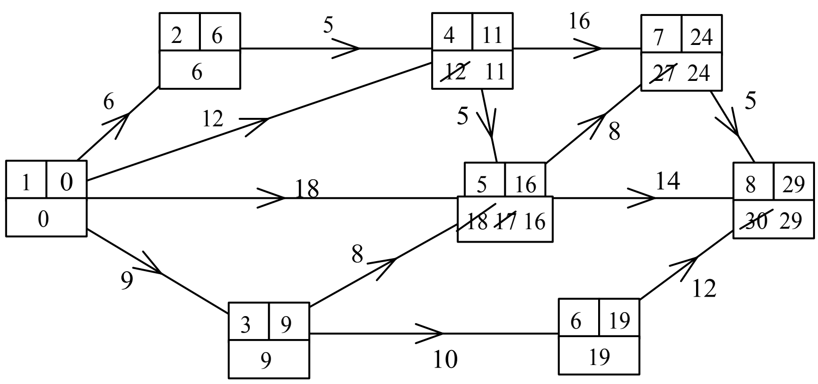


B3 — 7

4. (i) Nodes with odd numbers of arcs attached must be joined together, in pairs, so that all nodes become even. The pairing which gives the minimum extra length is then used. It is then possible to start and finish at any point — B2

(ii) Odd nodes are A, B, D and F — B1
AB + DF = 18 AD + BF = 23 AF + BD = 23 — M1 A1
Therefore, repeat sections AB and DF. — A1
e.g. P E F G C B D F D A B A P, with length 100 + 18 = 118 — M1 A1 — 8

5. Labelling : — M1 M1 M1

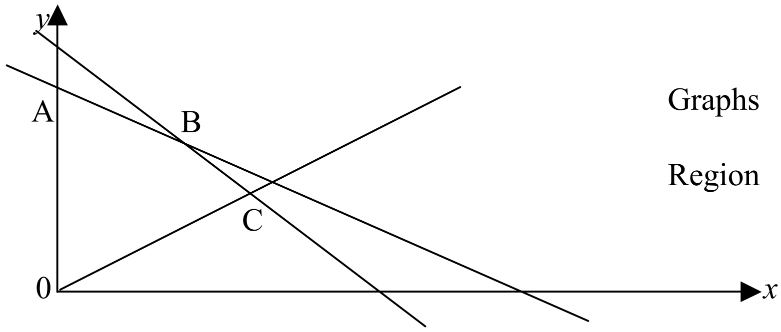


A1 A1

The arcs on the shortest route are those for which the difference in labelling at each end is equal to the length of the arc — B2

This gives the shortest route as ABCEFH, of length 29 — A1 A1 — 9

6. (i) Graphs and region



Graphs B1 B1 B1 B1

Region B1 B1

(ii) Co-ordinates of vertices are O (0, 0), A (0, 6) , B($^2/_3$, 4 $^1/_3$), C (1, 3) A1 A1 A1
This gives values of P as 0, 30, 24 $^1/_3$ and 19 respectively. Therefore, M1
maximum value of P is 30, when x = 0 and y = 6 A1 A1 12

7. (i)

P	x	y	z	r	s	
1	-2	-1	-3	0	0	0
0	1	1	2	1	0	6
0	2	5	1	0	1	10

M1 A1 A1

P	x	y	z	r	s	
1	0	4	-2	0	1	10
0	0	-1.5	1.5	1	-0.5	1
0	1	2.5	0.5	0	0.5	5

M1 A1 A1

P	x	y	z	r	s	
1	0	2	0	1 $^1/_3$	$^1/_3$	11 $^1/_3$
0	0	-1	1	$^2/_3$	-$^1/_3$	$^2/_3$
0	1	3	0	-$^1/_3$	$^2/_3$	4 $^2/_3$

M1 A1 A1

(ii) Max P = 11 $^1/_3$, when x = 4 $^2/_3$, y = 0 and z = $^2/_3$ A1 A1

(iii) Top row elements are all positive, so any increase in y, r or s will make P smaller; therefore solution is optimal B2 13