1. The interim stage of a Simplex calculation (in which *x, y, z, r* and *s* are all non-negative) is

P	*x*	*y*	*z*	*r*	*s*	
1	–2	0	0.6	$1\,{}^{1}/_{3}$	4	24
0	3	0	6.6	4	5.4	60
0	1	1	2.2	$-{}^{1}/_{6}$	4.8	30

(i) Perform one further iteration, and write down the maximum value of *P*, together with the corresponding values of *x, y* and *z*. [4]

(ii) Explain why your solution for *P* is a maximum. [2]

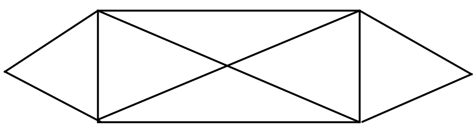
2. Over the Christmas holiday season, a number of TV programmes need to be recorded on video tapes. Each tape can take 180 minutes of programmes.

(i) Find the minimum number of tapes that would be needed to record programmes of length 35, 75, 90, 30, 30 120, 45, 80 and 45 minutes. [2]

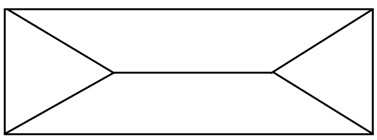
(ii) Use the first fit algorithm to fit the programmes onto tapes. Explain why the first-fit decreasing algorithm would be inappropriate in this situation. [4]

3. (i) For each of the following graphs, determine whether it is Eulerian, semi-Eulerian or neither.

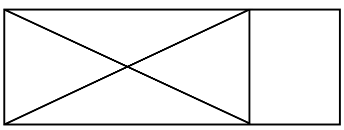
(a)



(b)



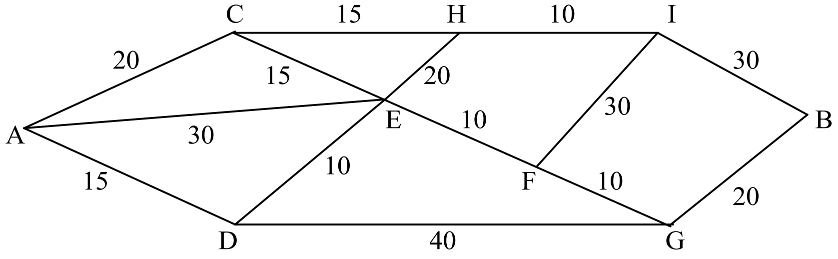
(c)



[4]

(ii) Explain the relevance of part (i) to the Route Inspection Problem, where each arc of a network must be traversed just once. State which type of graph allows the problem to be solved starting at any node. [3]

4. A computer package is being designed to find the quickest route between two towns, A and B, as shown. The time for each journey between towns is shown, in minutes.



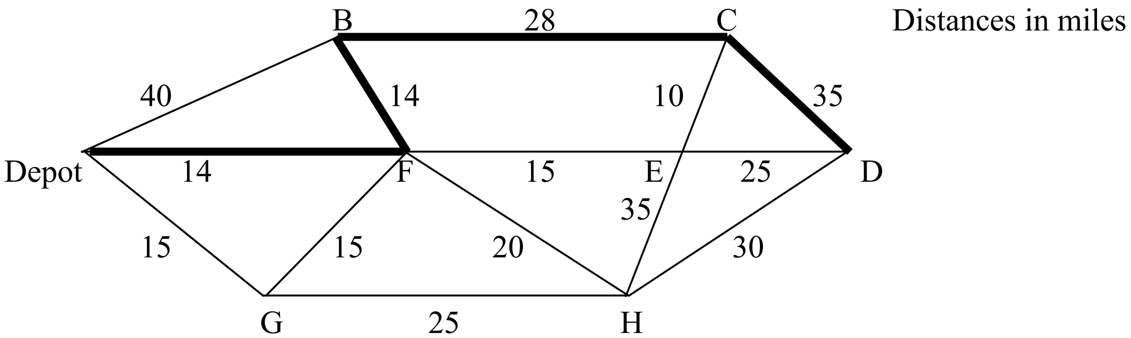
Turn over …

4. continued …
   (i) Assuming that towns themselves take no time to cross, use Dijkstra's algorithm to find the quickest route from A to B. [4]
   (ii) As a more realistic model, it is now assumed that each town (not including A and B) takes 30 minutes to cross. Determine whether this changes the result of part (i). [4]

5. A graph G has six vertices A, B, C, D, E and F, each connected to every other vertex.
   (i) Starting from A, show that Prim's algorithm for finding a minimum spanning tree for G requires 35 length inspections. [4]
   (ii) In general, Prim's algorithm is of order n^3. If a computer takes 0.5 seconds to find the minimum spanning tree for a network with 100 vertices, estimate how long it will take for a network with 500 vertices. [2]

   An alternative algorithm for finding the minimum spanning tree is to find the longest arc and delete it, so long as this does not separate the graph into two disjoint graphs. Then find the next longest and delete it, and so on. This process is continued until a spanning tree is formed.
   (iii) Calculate the number of length inspections this algorithm requires for the graph G, and hence compare it with Prim's algorithm. [4]

6. A delivery lorry has to visit all the outlets shown on this map. On motorways, indicated by the heavy lines, it can travel at 70 mph, whilst on the other roads it can only travel at 50 mph.



   (i) Use the nearest Neighbour algorithm to find an upper bound for the time taken to complete the journey, starting and finishing at the depot. [3]
   (ii) An alternative method for finding an upper bound is to find the minimum spanning tree, and double its length. Carry out this process, and explain why it is unlikely to give an exact solution to the problem of finding the minimum time. [4]
   (iii) Find a lower bound by deleting F. Explain why this is in fact an exact solution. [4]

7. A furniture warehouse needs to display chairs and settees. Chairs have a width of 1.2 m, while settees take up 3.2 m. There is a total aisle length of 250 m available, along which the chairs and settees are to be displayed side by side. At least two chairs should be displayed for each settee, but no more than 150 chairs should be displayed altogether. The profits on a chair and a settee are £50 and £80 respectively.
   (i) Write down inequalities for x and y, the numbers of chairs and settees respectively, to model the constraints, and write down the profit function. [3]
   (ii) Draw graphs to represent the inequalities. [3]
   (iii) Find the maximum profit that can be made when all the items in the warehouse are sold.

DECISION MATHS 1 (C) PAPER 2 : ANSWERS AND MARK SCHEME

1. (i)

P	x	y	z	r	s	
1	0	0	5	4	7.6	64
0	1	0	2.2	$1\,^1/_3$	1.8	20
0	0	1	0	-1.5	3	10

M1 A1 A1

Maximum $P = 64$, when $x = 20$, $y = 10$, $z = 0$ A1

(ii) $P = 64 - 5z - 4r - 7.6s$ means that any increase in these variables reduces P
Thus it is a maximum. B2 6

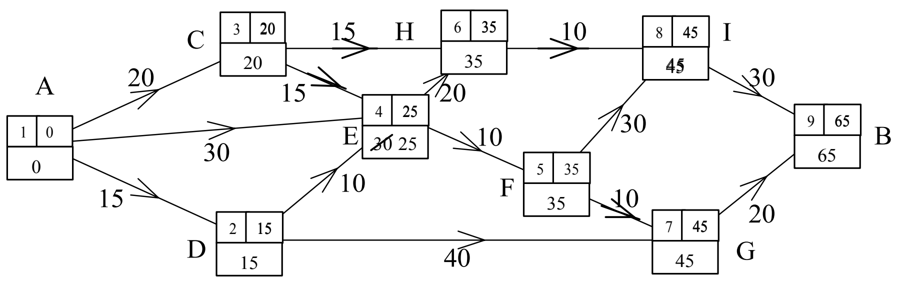
2. (i) Total time = 35 + 75 + 90 + 30 + 30 + 120 + 45 + 80 + 45 = 550
So minimum number of tapes = 550/180 = $3\,^1/_{18}$ i.e. need at least 4 M1 A1

(ii) Tape 1 : 35, 75, 30, 30
Tape 2 : 90, 45, 45
Tape 3 : 120
Tape 4 : 80 M1 A1 A1
The programmes cannot be sorted into decreasing order – they have to be recorded when they happen B1 6

3. (i) (a) is Eulerian, (b) is neither, (c) is Semi-Eulerian M1 A1 A1 A1

(ii) An odd node must occur (if at all) at the beginning or end of a path. Thus a traversable graph must not have more than two odd nodes. If it has exactly two (semi-Eulerian), one must be the start and the other the finish of the path. If all nodes are even (Eulerian), then the path can start and finish at any node B1 B1 B1 7

4.



(i) Labelling M1 A1 A1
Shortest path (by backtracking) : ADEFGB, time 65 minutes A1

(ii) This path has four intermediate towns. so add on 4 x 30 = 120, to get 185 B1
AEFGB only has 3 towns, so takes 70 + 3 x 30 = 160 i.e. originally took M1
5 minutes longer, but now 25 minutes quicker A1 A1 8

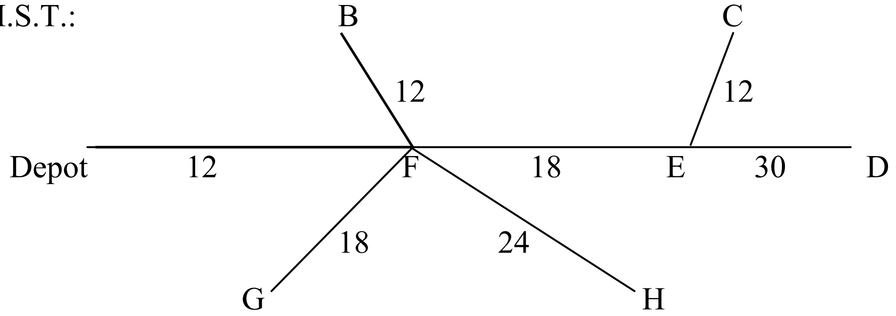
5. (i) 5 possibilities for first choice from A, then 2 x 4 choices from A and, say, B, then 3 x 3 from A, B and, say, C, then 4 x 2 then 5, giving total number of inspections required = 35 M1 A1 A1 A1

(ii) Ratio is 500/100 = 5, so time taken = original time x scale factor^3
$= 0.5 \times 5^3 = 62.5$ s M1 A1

(iii) Number of inspections = 15 + 14….. + 2 = 119 M1 A1 A1
This is many more than Prim requires, so the algorithm will take longer B1 10

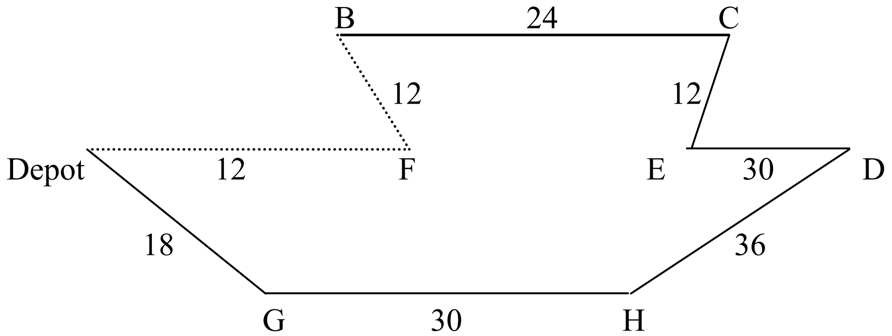
6. (i) Convert all distances to times in minutes (distance/speed), then get M1
Depot – F – B – C – E – D – H – G – Depot, taking 174 mins M1 A1

(ii) M.S.T.:



Length = 126, so upper bound is 252 M1 A1 A1
This will involve repeat visits to each town, so it is likely to be longer than necessary B1

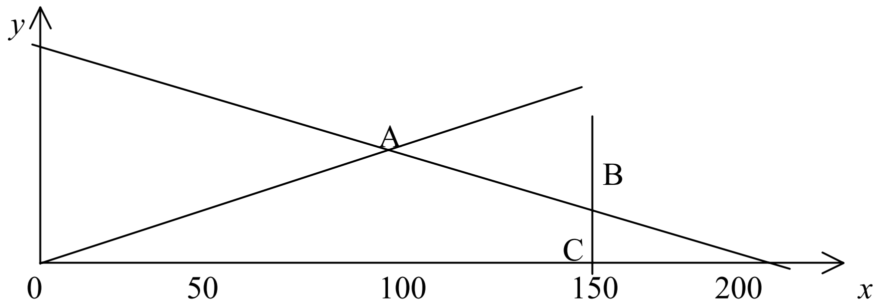
(iii) M.S.T. for remaining towns, plus F, is



Total = 12 + 12 + 150 = 174 M1 A1 A1
F joins the M.S.T. at its ends, to form a cycle. Since this is a practical path through every vertex, it is an exact solution B1 11

7. (i) $1.2x + 3.2y \leq 250$ i.e. $3x + 8y \leq 625$ B1
$x \geq 2y$, $x \leq 150$, $x, y \geq 0$ B1
$P = 50x + 80y$ B1

(ii)



B1 B1 B1

(iii) At O (0, 0) $P = 0$ At A (89.3, 44.6) $P = 8032$ M1 A1
At B (150, 21.875) $P = 9250$ At C (150, 0) $P = 7500$ M1 A1
Therefore, max profit at B; but we need integer solutions, so consider
(150, 21) : $P = 9180$ and (149, 22) : Max. $P = £9210$ M1 A1 12

