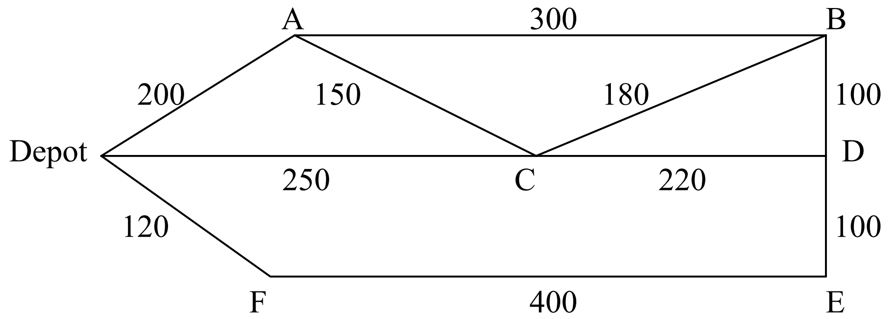
DECISION MATHEMATICS (C) UNIT 1 TEST PAPER 1

1. A simple graph G has 4 vertices, each of order d.
   (i) State the possible values of d. [2]
   (ii) If G is a connected graph, state which values of d are now possible. For each of these values, state the total number of arcs of the graph, and sketch the graph. [2]
   (iii) If G is also Eulerian, state the value of d. [1]

2. In a linear programming problem, the function $P = 2x + 3y$ is to be maximised, subject to the constraints $x + 3y \leq 7$, $9x + 8y \leq 36$, together with $x \geq 0$, $y \geq 0$.
   (i) Draw a graph to illustrate these constraints, showing the feasible region. [3]
   (ii) Find the maximum value of P, given that x and y must be integers. [3]

3. The diagram shows a network of roads, with the length of each in metres. A postman needs to deliver letters along each road.



   (i) Assuming that he delivers to both sides of a road as he walks along, use a suitable algorithm to find the minimum distance he must travel, starting and finishing at the depot. [5]
   (ii) If, instead, he delivers separately to each side of every road, find the distance that he must now travel. [3]

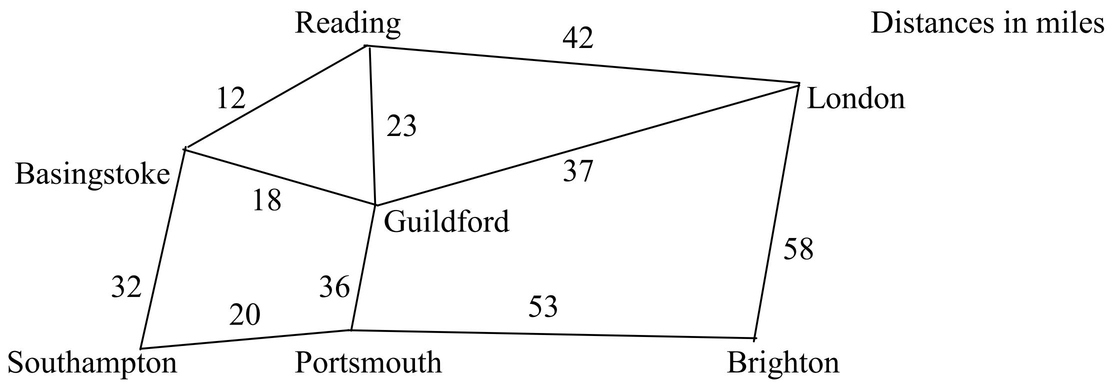
4. (i) Use Prim's algorithm, starting from A, to find the minimum spanning tree for the network described by this matrix:

	A	B	C	D	E	F
A	-	95	56	66	32	47
B	95	-	38	60	19	65
C	56	38	-	45	49	73
D	66	60	45	-	86	46
E	32	19	49	86	-	22
F	47	65	73	46	22	-

   Indicate clearly the order in which you select the arcs. [4]

(ii) Draw a diagram showing the minimum spanning tree, and state its length. [4]

5. For an election tour, the Prime Minister has to visit each town shown on the map, starting and finishing at London.



(i) By finding the minimum spanning tree of this network, give an upper bound for the length of his journey. [4]

(ii) Find a lower bound for the length of the journey by deleting Southampton. Explain why your solution is a lower bound. [4]

6. (i) Use the Bubble Sort algorithm to sort this list of numbers into ascending order, giving the state of the list after each rearrangement :

7 9 13 5 8 17 6 [3]

(ii) Give the total number of comparisons and swaps that have been made. [4]

(iii) Calculate the maximum number of comparisons and of swaps that might be needed when sorting a list of seven items using the Bubble Sort. [2]

(iv) Find the maximum number of comparisons and swaps that might be needed for a list of n items using the Bubble Sort. [3]

7. It is required to find the maximum value of $P = 2x + 4y + 5z$, given that $x + 3y + z \leq 5$ and $2x + y + 2z \leq 6$, together with $x, y, z \geq 0$.

(i) Write the constraints as equations involving slack variables r and s. [1]

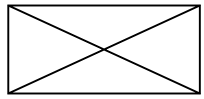
(ii) Use the Simplex Algorithm to find the maximum value of P. Write down the corresponding values of x, y and z. [10]

(iii) Explain how you know that your final tableau is optimal. [2]

DECISION MATHS 1 (C) PAPER 1 : ANSWERS AND MARK SCHEME

1. (i) $d = 0, 1, 2, 3$ — B2

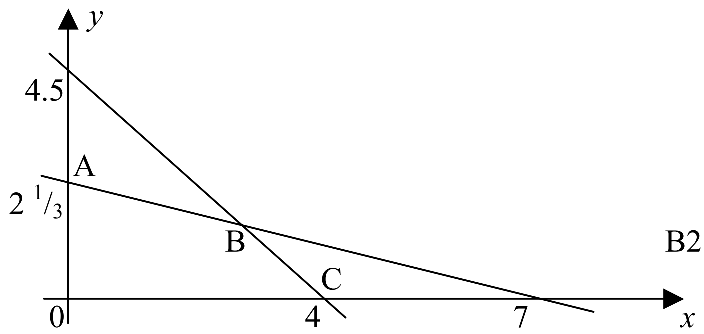
(ii) $d = 2$, giving 4 edges : and $d = 3$, with 6 edges :



B1 B1

(iii) To be Eulerian, all nodes must be even, so $d = 2$ — B1 — 5

2. (i) Graphs, intersection



B2

B = $(2\ {}^{14}/_{19},\ 1\ {}^{8}/_{19})$ — B1

(ii) $P(0) = 0$, $P(\text{A}) = 7$, $P(\text{B}) = 9\ {}^{14}/_{19}$, $P(\text{C}) = 8$ — M1

Integer points nearest B are (2, 1), (2, 2) and (3, 1). Only (2, 1) is within the region; this gives $P = 7$. Thus C is the best solution, giving $P = 8$ — M1 A1 — 6

3. (i) To make a traversable network, all odd nodes must be eliminated, by joining them to get even nodes. The odd nodes are A, B, D and Depot:

A - B + D - Depot	= 300 + 470 = 770	A1
A - D + B - Depot	= 370 + 430 = 800	A1
A - Depot + B - D	= 200 + 100 = 300	A1

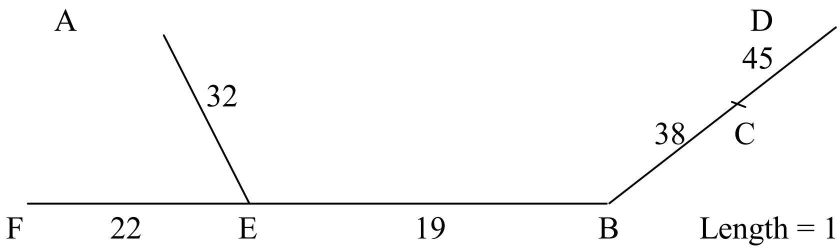
So he should repeat A - Depot and B - D — M1

This gives a total distance of 2020 + 300 = 2320 — A1

(ii) To cover every road twice, simply do 2 x 2020 = 4040, since every node is then even, and the network is traversable — M1 A1, A1 — 8

4. (i) Matrix method; order of selection A E B F C D — M1 M1 A1 A1

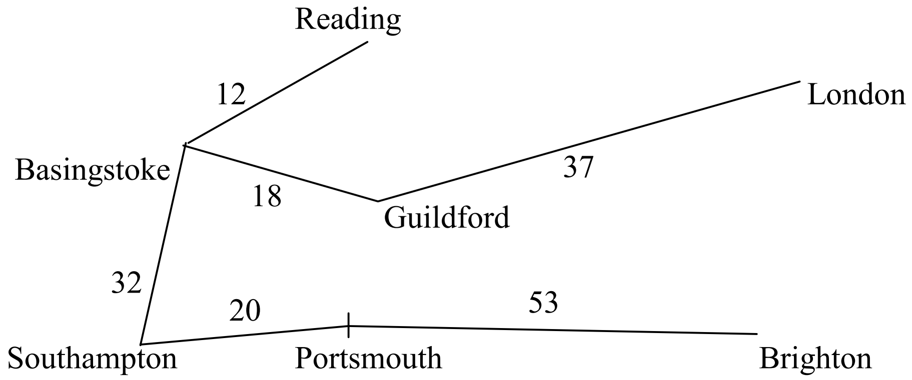
(ii)



Length = 156

M1 A1, A1, A1 — 8

5. (i) M.S.T. :



M1 A1

Length = 172, so upper bound is 344 miles — M1 A1

(ii) The new M.S.T. has length 12 + 18 + 36 + 37 + 53 = 156 — M1 A1
With Southampton added on, total is 20 + 32 + 156 = 208 — A1
This is lower bound because to link all the non-Southampton towns needs at least 156 miles, and to join Southampton must take at least another 52 — B1 — 8

6. (i)

7	9	5	8	13	6	17
7	5	8	9	6	13	17
5	7	8	6	9	13	17
5	7	6	8	9	13	17
5	6	7	8	9	13	17

M1 A1 A1

(ii) Comparisons: 6 + 5 + 4 + 3 + 2 + 1 = 21; swaps 3 + 3 + 2 + 1 + 1 = 10 — M1 A1 M1 A1
(iii) Comps: 21, as before; swaps: 21 (if each comparison results in a swap) — B1 B1
(iv) $(n-1) + (n-2) + \ldots + 2 + 1 = n(n-1)/2$ comparisons, and the same number of swaps — M1 M1 A1 — 12

7. (i) $x + 3y + z + r = 5$ and $2x + y + 2z + s = 6$ — B1
(ii)

P	x	y	z	r	s	
1	–2	–4	–5	0	0	0
0	1	3	1	1	0	5
0	2	1	2	0	1	6

M1 M1 A1

1	3	–1.5	0	0	2.5	15
0	0	2.5	0	1	–0.5	2
0	1	0.5	1	0	0.5	3

M1 M1 A1

1	3	0	0	0.6	2.2	16.2
0	0	1	0	0.4	–0.2	0.8
0	1	0	1	–0.2	0.6	2.6

So max. value of P is 16.2, when $x = r = s = 0$, $y = 0.8$ and $z = 2.6$ — A1 A1 A1 A1
(iii) All entries in objective function row are positive, so this can be written as $P = 16.2 - 3x - 0.6r - 2.2s$; thus, any increase in x, r or s will decrease — B1
P, so it is a maximum — B1 — 13