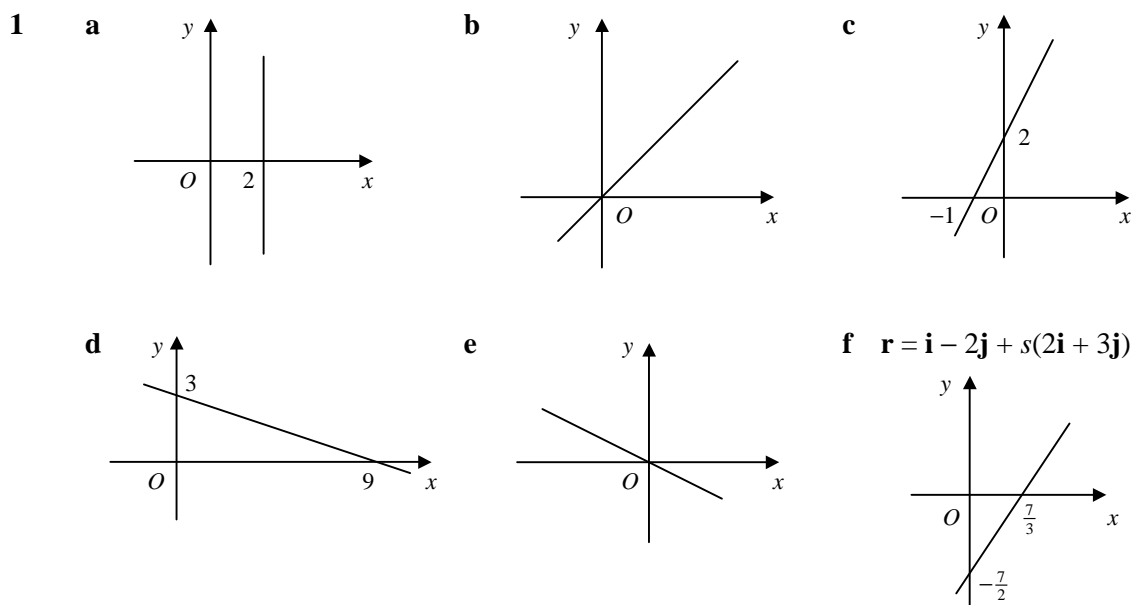


C4 VECTORS

Answers - Worksheet C

Note: For this worksheet especially, there may be alternative correct answers



2 a $\mathbf{r} = -\mathbf{i} + \mathbf{j} + s(3\mathbf{i} - 2\mathbf{j})$

b $\mathbf{r} = 4\mathbf{j} + s\mathbf{i}$

c $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + s(\mathbf{i} + 5\mathbf{j})$

3 a $\text{dir}^n = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\therefore \mathbf{r} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

b $\text{dir}^n = \begin{pmatrix} -1 \\ 1 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$\therefore \mathbf{r} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} + s \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

c $\text{dir}^n = \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

$\therefore \mathbf{r} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} + s \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

4 a $-1 + 2\lambda = 5 \therefore \lambda = 3$

$3 + c\lambda = 3 + 3c = 0 \therefore c = -1$

b $c\mathbf{i} + 2\mathbf{j} = k(6\mathbf{i} + 3\mathbf{j})$

$\therefore k = \frac{2}{3}$

$\therefore c = 4$

5 a $\mathbf{r} = -\mathbf{i} + s\mathbf{j}$

b $\mathbf{r} = s(\mathbf{i} + 2\mathbf{j})$

c $\mathbf{r} = \mathbf{j} + s(\mathbf{i} + 3\mathbf{j})$

d $\mathbf{r} = -2\mathbf{j} + s(4\mathbf{i} + 3\mathbf{j})$

e $\mathbf{r} = 5\mathbf{j} + s(2\mathbf{i} - \mathbf{j})$

f $y = \frac{1}{4}x + 2$

$\therefore \mathbf{r} = 2\mathbf{j} + s(4\mathbf{i} + \mathbf{j})$

6 a $x = 2 + 3\lambda, y = 1 + 2\lambda$

b $\lambda = \frac{x-2}{3} = \frac{y-1}{2}$

$2(x-2) = 3(y-1)$

$2x - 4 = 3y - 3$

$2x - 3y - 1 = 0$

- 7**
- a** $x = 3 + \lambda, y = 2\lambda$
 $\lambda = x - 3 = \frac{y}{2}$
 $2(x - 3) = y$
 $2x - y - 6 = 0$
- b** $x = 1 + 3\lambda, y = 4 + \lambda$
 $\lambda = \frac{x-1}{3} = y - 4$
 $x - 1 = 3(y - 4)$
 $x - 3y + 11 = 0$
- c** $x = 4\lambda, y = 2 - \lambda$
 $\lambda = \frac{x}{4} = 2 - y$
 $x = 4(2 - y)$
 $x + 4y - 8 = 0$
- d** $x = -2 + 5\lambda, y = 1 + 2\lambda$
 $\lambda = \frac{x+2}{5} = \frac{y-1}{2}$
 $2(x + 2) = 5(y - 1)$
 $2x - 5y + 9 = 0$
- e** $x = 2 - 3\lambda, y = -3 + 4\lambda$
 $\lambda = \frac{x-2}{-3} = \frac{y+3}{4}$
 $4(x - 2) = -3(y + 3)$
 $4x + 3y + 1 = 0$
- f** $x = \lambda + 3, y = -2\lambda - 1$
 $\lambda = x - 3 = \frac{y+1}{-2}$
 $-2(x - 3) = y + 1$
 $2x + y - 5 = 0$
- 8**
- a** $\begin{pmatrix} 3 \\ -1 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} -6 \\ 2 \end{pmatrix}$
 \therefore parallel
 $(1, 2)$ lies on first line
when $x = 1$ on second line
 $-2 - 6t = 1 \Rightarrow t = -\frac{1}{2}$
 $\Rightarrow y = 3 + 2(-\frac{1}{2}) = 2$
parallel and common point
 \therefore identical
- b** $\begin{pmatrix} 1 \\ 4 \end{pmatrix} \neq k \begin{pmatrix} 4 \\ 1 \end{pmatrix}$
 \therefore not parallel
- c** $\begin{pmatrix} 2 \\ 4 \end{pmatrix} = \frac{2}{3} \begin{pmatrix} 3 \\ 6 \end{pmatrix}$
 \therefore parallel
 $(2, -5)$ lies on first line
when $x = 2$ on second line
 $-1 + 3t = 2 \Rightarrow t = 1$
 $\Rightarrow y = 1 + 6(1) = 7$
 $\therefore (2, -5)$ not on second line
 \therefore parallel but not identical
- 9**
- a** $1 + \lambda = 2 + 3\mu$ (1)
 $2 = 1 + \mu$ (2)
(2) $\Rightarrow \mu = 1$
 $\therefore 5\mathbf{i} + 2\mathbf{j}$
- b** $4 - \lambda = 5 + 2\mu$ (1)
 $1 + \lambda = -2 - 3\mu$ (2)
(1) + (2) $\Rightarrow 5 = 3 - \mu$
 $\mu = -2$
 $\therefore \mathbf{i} + 4\mathbf{j}$
- c** $2\lambda = 2 - \mu$ (1)
 $1 - \lambda = 10 + 3\mu$ (2)
(1) + 2×(2) $\Rightarrow 2 = 22 + 5\mu$
 $\mu = -4$
 $\therefore 6\mathbf{i} - 2\mathbf{j}$
- d** $-1 - 4\lambda = 2 - \mu$ (1)
 $5 + 6\lambda = -2 + 2\mu$ (2)
 $2 \times (1) + (2) \Rightarrow 3 - 2\lambda = 2$
 $\lambda = \frac{1}{2}$
 $\therefore -3\mathbf{i} + 8\mathbf{j}$
- e** $-2 - 3\lambda = -3 + 5\mu$ (1)
 $11 + 4\lambda = -7 + 3\mu$ (2)
 $4 \times (1) + 3 \times (2)$
 $\Rightarrow 25 = -33 + 29\mu$
 $\mu = 2$
 $\therefore 7\mathbf{i} - \mathbf{j}$
- f** $1 + 3\lambda = 3 + \mu$ (1)
 $2 + 2\lambda = 5 + 4\mu$ (2)
 $2 \times (1) - 3 \times (2)$
 $\Rightarrow -4 = -9 - 10\mu$
 $\mu = -\frac{1}{2}$
 $\therefore \frac{5}{2}\mathbf{i} + 3\mathbf{j}$
- 10**
- a** $\mathbf{r} = 4\mathbf{i} + \mathbf{k} + s(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$
- b** $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + s\mathbf{k}$
- c** $\mathbf{r} = -\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + s(2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$
- 11**
- a** $\overline{AB} = (6\mathbf{i} - 3\mathbf{j} + \mathbf{k}) - (5\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = \mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$
- b** $\mathbf{r} = (5\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + s(\mathbf{i} - 4\mathbf{j} + 3\mathbf{k})$
- c** $5 + s = 3 \Rightarrow s = -2$
when $s = -2$, $\mathbf{r} = (5\mathbf{i} + \mathbf{j} - 2\mathbf{k}) - 2(\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) = 3\mathbf{i} + 9\mathbf{j} - 8\mathbf{k}$
 $\therefore l$ passes through $(3, 9, -8)$

- 12** a direction = $(5\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}) - (\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$
 $= 4\mathbf{i} + \mathbf{j} + 2\mathbf{k}$
 $\therefore \mathbf{r} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + s(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$
- b direction = $(\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}) - (3\mathbf{i} - 2\mathbf{k})$
 $= -2\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$
 $\therefore \mathbf{r} = 3\mathbf{i} - 2\mathbf{k} + s(-2\mathbf{i} + 5\mathbf{j} + 4\mathbf{k})$
- c $\mathbf{r} = s(6\mathbf{i} - \mathbf{j} + 2\mathbf{k})$
- d direction = $(4\mathbf{i} - 7\mathbf{j} + \mathbf{k}) - (-\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$
 $= 5\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$
 $\therefore \mathbf{r} = -\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + s(5\mathbf{i} - 5\mathbf{j} - 2\mathbf{k})$
- 13** a $3 + 2\lambda = 9 \therefore \lambda = 3$
 $-5 + a\lambda = -5 + 3a = -2 \therefore a = 1$
 $1 + b\lambda = 1 + 3b = -8 \therefore b = -3$
- b $2\mathbf{i} + a\mathbf{j} + b\mathbf{k} = k(8\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$
 $\therefore k = \frac{1}{4}$
 $\therefore a = -1, b = \frac{1}{2}$
- 14** a $x = 2 + 3\lambda,$
 $y = 3 + 5\lambda,$
 $z = 2\lambda,$
 $(\lambda =) \frac{x-2}{3} = \frac{y-3}{5} = \frac{z}{2}$
- b $x = 4 + \lambda,$
 $y = -1 + 6\lambda,$
 $z = 3 + 3\lambda,$
 $(\lambda =) x - 4 = \frac{y+1}{6} = \frac{z-3}{3}$
- c $x = -1 + 4\lambda,$
 $y = 5 - 2\lambda,$
 $z = -2 - \lambda,$
 $(\lambda =) \frac{x+1}{4} = \frac{y-5}{-2} = \frac{z+2}{-1}$
- 15** a $s = \frac{x-1}{3} = \frac{y+4}{2} = z - 5$
 $x = 1 + 3s,$
 $y = -4 + 2s,$
 $z = 5 + s,$
 $\mathbf{r} = \mathbf{i} - 4\mathbf{j} + 5\mathbf{k} + s(3\mathbf{i} + 2\mathbf{j} + \mathbf{k})$
- b $s = \frac{x}{4} = \frac{y-1}{-2} = \frac{z+7}{3}$
 $x = 4s,$
 $y = 1 - 2s,$
 $z = -7 + 3s,$
 $\mathbf{r} = \mathbf{j} - 7\mathbf{k} + s(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$
- c $s = \frac{x+5}{-4} = y + 3 = z$
 $x = -5 - 4s,$
 $y = -3 + s,$
 $z = s,$
 $\mathbf{r} = -5\mathbf{i} - 3\mathbf{j} + s(-4\mathbf{i} + \mathbf{j} + \mathbf{k})$
- 16** $4 + s = 7 - 3t$ (1)
 $-2s = 2 + 2t$ (2)
 $3 + 2s = -5 + t$ (3)
 $(2) + (3) \Rightarrow 3 = -3 + 3t$
 $t = 2, s = -3$
 check (1) $4 + (-3) = 7 - 3(2)$
 true \therefore intersect
 point of intersection: $(1, 6, -3)$
- 17** $2 + \lambda = 1 + \mu$ (1)
 $-1 + \lambda = 4 - 2\mu$ (2)
 $4 + 3\lambda = 3 + \mu$ (3)
 $(1) - (2) \Rightarrow 3 = -3 + 3\mu$
 $\mu = 2, \lambda = 1$
 check (3) $4 + 3(1) = 3 + (2)$
 false \therefore do not intersect
 $(\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \neq k(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \therefore$ not parallel
 \therefore skew

- 18 a** $3 + 4\lambda = 3 + \mu$ (1)
- $$1 + \lambda = 2$$
- (2)
-
- $5 - \lambda = -4 + 2\mu$
- (3)
-
- (2)
- $\Rightarrow \lambda = 1$
-
- sub. (1)
- $\mu = 4$
-
- check (3)
- $5 - (1) = -4 + 2(4)$
-
- true
- \therefore
- intersect
- position vector of intersection: $\begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix}$
- c** $8 + \lambda = -2 + 4\mu$ (1)
 $2 + 3\lambda = 2 - 3\mu$ (2)
 $-4 - 2\lambda = 8 - 4\mu$ (3)
 (1) + (3) $\Rightarrow 4 - \lambda = 6$
 $\lambda = -2, \mu = 2$
 check (2) $2 + 3(-2) = 2 - 3(2)$
 true \therefore intersect
- position vector of intersection: $\begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$
- e** $4 + 2\lambda = 3 + 5\mu$ (1)
 $-1 + 5\lambda = -2 - 3\mu$ (2)
 $3 - 3\lambda = 1 - 4\mu$ (3)
 $3 \times (1) + 2 \times (3) \Rightarrow 18 = 11 + 7\mu$
 $\mu = 1, \lambda = 2$
 check (2) $-1 + 5(2) = -2 - 3(1)$
 false \therefore do not intersect
- $$\begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} \neq k \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix}$$
- \therefore skew
- b** $\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} -4 \\ 2 \\ 6 \end{pmatrix}$
 \therefore parallel
- d** $1 + \lambda = 7 + 2\mu$ (1)
 $5 + 4\lambda = -6 + \mu$ (2)
 $2 - 2\lambda = -5 - 3\mu$ (3)
 $2 \times (1) + (3) \Rightarrow 4 = 9 + \mu$
 $\mu = -5, \lambda = -4$
 check (2) $5 + 4(-4) = -6 + (-5)$
 true \therefore intersect
- position vector of intersection: $\begin{pmatrix} -3 \\ -11 \\ 10 \end{pmatrix}$
- f** $6\lambda = -12 + 5\mu$ (1)
 $7 - 4\lambda = -1 + 2\mu$ (2)
 $-2 + 8\lambda = 11 - 3\mu$ (3)
 $2 \times (2) + (3) \Rightarrow 12 = 9 + \mu$
 $\mu = 3, \lambda = \frac{1}{2}$
 check (1) $6(\frac{1}{2}) = -12 + 5(3)$
 true \therefore intersect
- position vector of intersection: $\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$