

C4 VECTORS

Answers - Worksheet B

- 1** **a** $6\mathbf{i} + \mathbf{j}$ **b** $-4\mathbf{i} + 2\mathbf{j}$ **c** $-6\mathbf{i}$ **d** $10\mathbf{i} - 2\mathbf{j}$
- 2** **a** $= 4(\mathbf{i} - 3\mathbf{j})$
 $= 4\mathbf{i} - 12\mathbf{j}$
c $= 2(\mathbf{i} - 3\mathbf{j}) + 3(4\mathbf{i} + 2\mathbf{j})$
 $= 14\mathbf{i}$
- 3** **a** $= \sqrt{9+16} = 5$
c $\mathbf{p} + 2\mathbf{q} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$
 $|\mathbf{p} + 2\mathbf{q}| = 5$
- 4** **a** $= \tan^{-1} \frac{1}{2} = 26.6^\circ$
c $5\mathbf{p} + \mathbf{q} = 5(2\mathbf{i} + \mathbf{j}) + (\mathbf{i} - 3\mathbf{j}) = 11\mathbf{i} + 2\mathbf{j}$
angle $= \tan^{-1} \frac{2}{11} = 10.3^\circ$
- 5** **a** $\left| \begin{pmatrix} 4 \\ 3 \end{pmatrix} \right| = \sqrt{16+9} = 5$
 $\therefore \frac{1}{5} \begin{pmatrix} 4 \\ 3 \end{pmatrix}$
c $\left| \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right| = \sqrt{1+1} = \sqrt{2}$
 $\therefore \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \frac{1}{2}\sqrt{2} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$
- 6** **a** $|5\mathbf{i} + 12\mathbf{j}| = \sqrt{25+144} = 13$
 $\therefore \frac{26}{13} (5\mathbf{i} + 12\mathbf{j}) = 10\mathbf{i} + 24\mathbf{j}$
b $|-6\mathbf{i} - 8\mathbf{j}| = \sqrt{36+64} = 10$
 $\therefore \frac{15}{10} (-6\mathbf{i} - 8\mathbf{j}) = -9\mathbf{i} - 12\mathbf{j}$
c $|2\mathbf{i} - 4\mathbf{j}| = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$
 $\therefore \frac{5}{2\sqrt{5}} (2\mathbf{i} - 4\mathbf{j}) = \sqrt{5} (\mathbf{i} - 2\mathbf{j})$
- 7** **a** $(2\mathbf{i} + \lambda\mathbf{j}) + (\mu\mathbf{i} - 5\mathbf{j}) = 3\mathbf{i} - \mathbf{j}$
 $2 + \mu = 3$ and $\lambda - 5 = -1$
 $\therefore \lambda = 4, \mu = 1$
- 8** **a** $6\mathbf{i} + c\mathbf{j} = 3(2\mathbf{i} + \mathbf{j})$
 $\therefore c = 3$
c $36 + c^2 = 10^2 = 100$
 $\therefore c^2 = 64$
 $c > 0 \therefore c = 8$
- b** $2(2\mathbf{i} + \lambda\mathbf{j}) - (\mu\mathbf{i} - 5\mathbf{j}) = -3\mathbf{i} + 8\mathbf{j}$
 $4 - \mu = -3$ and $2\lambda + 5 = 8$
 $\therefore \lambda = \frac{3}{2}, \mu = 7$
- b** $6\mathbf{i} + c\mathbf{j} = -\frac{2}{3}(-9\mathbf{i} - 6\mathbf{j})$
 $\therefore c = 4$
d $36 + c^2 = (3\sqrt{5})^2 = 45$
 $\therefore c^2 = 9$
 $c > 0 \therefore c = 3$

9 a $a(\mathbf{i} + 3\mathbf{j}) + b(4\mathbf{i} - 2\mathbf{j}) = -5\mathbf{i} + 13\mathbf{j}$

$$\therefore a + 4b = -5 \quad (1)$$

$$\text{and } 3a - 2b = 13 \quad (2)$$

$$(1) + 2 \times (2) \Rightarrow 7a = 21$$

$$\therefore a = 3, b = -2$$

b $c(\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - 2\mathbf{j}) = k\mathbf{j}$

$$\therefore c + 4 = 0$$

$$\therefore c = -4$$

c $(\mathbf{i} + 3\mathbf{j}) + d(4\mathbf{i} - 2\mathbf{j}) = k(3\mathbf{i} - \mathbf{j})$

$$\therefore 1 + 4d = 3k$$

$$\text{and } 3 - 2d = -k$$

$$(1) + 2 \times (2) \Rightarrow 7 = k$$

$$\therefore d = 5$$

10 a $\overrightarrow{AB} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} -8 \\ -4 \end{pmatrix}$

b $|\overrightarrow{AB}| = \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5}$

c $= \overrightarrow{OA} + \frac{1}{2} \overrightarrow{AB}$

$$= \begin{pmatrix} 3 \\ 6 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -8 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

d $\overrightarrow{OC} = \overrightarrow{AB}$

$$\therefore \text{pos. vector} = \begin{pmatrix} -8 \\ -4 \end{pmatrix}$$

11 a $= \sqrt{(2-4)^2 + (-3-0)^2 + (3-9)^2}$
 $= \sqrt{4+9+36}$
 $= 7$

b $= \sqrt{(7-11)^2 + (-1+3)^2 + (3-5)^2}$
 $= \sqrt{16+4+4}$
 $= \sqrt{24} = 2\sqrt{6} = 4.90 \text{ (3sf)}$

12 a $= \sqrt{16+4+16}$
 $= 6$

b $= \sqrt{1+1+1}$
 $= \sqrt{3} = 1.73 \text{ (3sf)}$

c $= \sqrt{64+1+16}$
 $= 9$

d $= \sqrt{9+25+1}$
 $= \sqrt{35} = 5.92 \text{ (3sf)}$

13 a $|5\mathbf{i} - 2\mathbf{j} + 14\mathbf{k}| = \sqrt{25+4+196} = 15$
 $\therefore \frac{1}{15}(5\mathbf{i} - 2\mathbf{j} + 14\mathbf{k})$

b $|2\mathbf{i} + 11\mathbf{j} - 10\mathbf{k}| = \sqrt{4+121+100} = 15$
 $\therefore \frac{10}{15}(2\mathbf{i} + 11\mathbf{j} - 10\mathbf{k}) = \frac{2}{3}(2\mathbf{i} + 11\mathbf{j} - 10\mathbf{k})$

c $|-5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}| = \sqrt{25+16+4} = \sqrt{45} = 3\sqrt{5}$
 $\therefore \frac{20}{3\sqrt{5}}(-5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = \frac{4}{3}\sqrt{5}(-5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$

14 $\lambda^2 + 144 + 16 = 14^2 = 196$
 $\lambda^2 = 36$
 $\lambda = \pm 6$

$$15 \quad \mathbf{a} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} + 2 \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ -1 \\ 1 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$$

$$\mathbf{c} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -3 \end{pmatrix}$$

$$\mathbf{d} = 2 \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} - 3 \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \\ -3 \end{pmatrix} = \begin{pmatrix} -12 \\ 17 \\ -8 \end{pmatrix}$$

$$16 \quad \mathbf{a} \quad -2\mathbf{i} + \lambda\mathbf{j} + \mu\mathbf{k} = -\frac{1}{2}(4\mathbf{i} + 2\mathbf{j} - 8\mathbf{k})$$

$$\therefore \lambda = -1, \mu = 4$$

$$\mathbf{b} \quad -2\mathbf{i} + \lambda\mathbf{j} + \mu\mathbf{k} = \frac{2}{5}(-5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k})$$

$$\therefore \lambda = 8, \mu = -4$$

$$17 \quad \mathbf{a} \quad 2\mathbf{p} - \mathbf{q} = 2(\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}) - (-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$$

$$= 3\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$$

$$\therefore |2\mathbf{p} - \mathbf{q}| = \sqrt{9 + 36 + 36} = 9$$

$$\mathbf{b} \quad (\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}) + k(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$$

$$= l(2\mathbf{i} - 4\mathbf{j} - 7\mathbf{k})$$

$$\therefore \quad 1 - k = 2l \quad (1)$$

$$-2 + 2k = -4l \quad (2)$$

$$4 + 2k = -7l \quad (3)$$

[(1) and (2) are the same equation]

$$(2) - (3) \Rightarrow -6 = 3l$$

$$\therefore l = -2$$

$$\therefore k = 5$$

$$19 \quad \mathbf{a} \quad (\lambda\mathbf{i} - 2\lambda\mathbf{j} + \mu\mathbf{k}) = k(2\mathbf{i} - 4\mathbf{j} - 3\mathbf{k})$$

$$\therefore \quad \lambda = 2k \quad (1)$$

$$-2\lambda = -4k \quad (2)$$

$$\mu = -3k \quad (3)$$

[(1) and (2) are the same equation]

$$3 \times (1) + 2 \times (3) \Rightarrow 3\lambda + 2\mu = 0$$

$$\mathbf{b} \quad \lambda^2 + (-2\lambda)^2 + \mu^2 = (2\sqrt{29})^2$$

$$5\lambda^2 + \mu^2 = 116$$

$$\mu = -\frac{3}{2}\lambda \Rightarrow 5\lambda^2 + \frac{9}{4}\lambda^2 = 116$$

$$\lambda^2 = 16$$

$$\lambda = \pm 4$$

$$\mu = -\frac{3}{2}\lambda \text{ and } \mu > 0 \therefore \lambda = -4, \mu = 6$$

$$21 \quad \mathbf{a} \quad d^2 = (9 - t)^2 + (1 + 2t)^2 + (5 - t)^2$$

$$= 81 - 18t + t^2 + 1 + 4t + 4t^2 + 25 - 10t + t^2$$

$$= 6t^2 - 24t + 107$$

$$\mathbf{b} \quad d^2 = 6(t^2 - 4t) + 107 = 6[(t - 2)^2 - 4] + 107$$

$$= 6(t - 2)^2 + 83$$

\therefore closest when $t = 2$

$$\text{min. } d = \sqrt{83} = 9.11 \text{ m (3sf)}$$

$$18 \quad \mathbf{a} \quad \overrightarrow{AB} = (-4\mathbf{i} + \mathbf{j} + 8\mathbf{k}) - (-2\mathbf{i} + 7\mathbf{j} + 4\mathbf{k})$$

$$= -2\mathbf{i} - 6\mathbf{j} + 4\mathbf{k}$$

$$\text{pos. vec of mid-point} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$$

$$= (-2\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}) + \frac{1}{2}(-2\mathbf{i} - 6\mathbf{j} + 4\mathbf{k})$$

$$= -3\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$$

$$\mathbf{b} \quad \overrightarrow{AC} = (6\mathbf{i} - 5\mathbf{j}) - (-2\mathbf{i} + 7\mathbf{j} + 4\mathbf{k})$$

$$= 8\mathbf{i} - 12\mathbf{j} - 4\mathbf{k}$$

$$\overrightarrow{AD} = \overrightarrow{OA} + \frac{3}{4}\overrightarrow{AC}$$

$$= (-2\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}) + \frac{3}{4}(8\mathbf{i} - 12\mathbf{j} - 4\mathbf{k})$$

$$= 4\mathbf{i} - 2\mathbf{j} + \mathbf{k}$$

$$20 \quad \mathbf{a} \quad \overrightarrow{BC} = \begin{pmatrix} 6 \\ 1 \\ -8 \end{pmatrix} - \begin{pmatrix} 12 \\ -7 \\ -4 \end{pmatrix} = \begin{pmatrix} -6 \\ 8 \\ -4 \end{pmatrix}$$

$$\overrightarrow{OM} = \overrightarrow{OB} + \frac{1}{2}\overrightarrow{BC}$$

$$= \begin{pmatrix} 12 \\ -7 \\ -4 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -6 \\ 8 \\ -4 \end{pmatrix} = \begin{pmatrix} 9 \\ -3 \\ -6 \end{pmatrix}$$

$$\mathbf{b} \quad \overrightarrow{OM} = \frac{3}{2}\overrightarrow{OA}$$

$\therefore \overrightarrow{OM}$ and \overrightarrow{OA} are parallel
common point O

$\therefore O, A$ and M are collinear