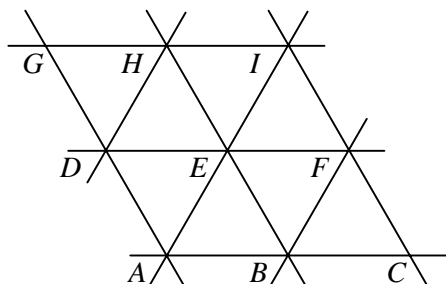


# C4 VECTORS

## Worksheet A

1

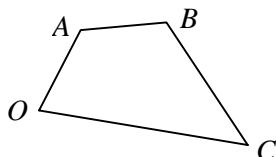


The diagram shows three sets of equally-spaced parallel lines.

Given that  $\overrightarrow{AC} = \mathbf{p}$  and that  $\overrightarrow{AD} = \mathbf{q}$ , express the following vectors in terms of  $\mathbf{p}$  and  $\mathbf{q}$ .

- a**  $\overrightarrow{CA}$       **b**  $\overrightarrow{AG}$       **c**  $\overrightarrow{AB}$       **d**  $\overrightarrow{DF}$       **e**  $\overrightarrow{HE}$       **f**  $\overrightarrow{AF}$   
**g**  $\overrightarrow{AH}$       **h**  $\overrightarrow{DC}$       **i**  $\overrightarrow{CG}$       **j**  $\overrightarrow{IA}$       **k**  $\overrightarrow{EC}$       **l**  $\overrightarrow{IB}$

2

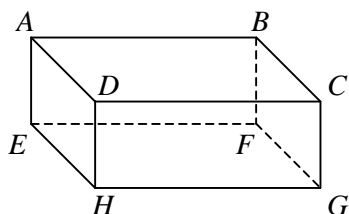


In the quadrilateral shown,  $\overrightarrow{OA} = \mathbf{u}$ ,  $\overrightarrow{AB} = \mathbf{v}$  and  $\overrightarrow{OC} = \mathbf{w}$ .

Find expressions in terms of  $\mathbf{u}$ ,  $\mathbf{v}$  and  $\mathbf{w}$  for

- a**  $\overrightarrow{OB}$       **b**  $\overrightarrow{AC}$       **c**  $\overrightarrow{CB}$

3

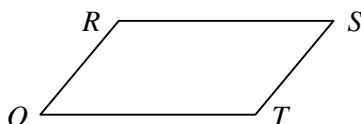


The diagram shows a cuboid.

Given that  $\overrightarrow{AB} = \mathbf{p}$ ,  $\overrightarrow{AD} = \mathbf{q}$  and  $\overrightarrow{AE} = \mathbf{r}$ , find expressions in terms of  $\mathbf{p}$ ,  $\mathbf{q}$  and  $\mathbf{r}$  for

- a**  $\overrightarrow{BC}$       **b**  $\overrightarrow{AF}$       **c**  $\overrightarrow{DE}$       **d**  $\overrightarrow{AG}$       **e**  $\overrightarrow{GB}$       **f**  $\overrightarrow{BH}$

4



The diagram shows parallelogram  $ORST$ .

Given that  $\overrightarrow{OR} = \mathbf{a} + 2\mathbf{b}$  and that  $\overrightarrow{OT} = \mathbf{a} - 2\mathbf{b}$ ,

**a** find expressions in terms of  $\mathbf{a}$  and  $\mathbf{b}$  for

- i**  $\overrightarrow{OS}$       **ii**  $\overrightarrow{TR}$

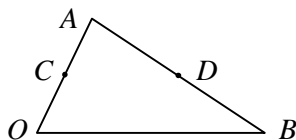
Given also that  $\overrightarrow{OA} = \mathbf{a}$  and that  $\overrightarrow{OB} = \mathbf{b}$ ,

**b** copy the diagram and show the positions of the points  $A$  and  $B$ .

## C4 VECTORS

## Worksheet A continued

5



The diagram shows triangle  $OAB$  in which  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$ .

The points  $C$  and  $D$  are the mid-points of  $OA$  and  $AB$  respectively.

**a** Find and simplify expressions in terms of  $\mathbf{a}$  and  $\mathbf{b}$  for

**i**  $\overrightarrow{OC}$       **ii**  $\overrightarrow{AB}$       **iii**  $\overrightarrow{AD}$       **iv**  $\overrightarrow{OD}$       **v**  $\overrightarrow{CD}$

**b** Explain what your expression for  $\overrightarrow{CD}$  tells you about  $\overrightarrow{OB}$  and  $\overrightarrow{CD}$ .

**6** Given that vectors  $\mathbf{p}$  and  $\mathbf{q}$  are not parallel, state whether or not each of the following pairs of vectors are parallel.

**a**  $2\mathbf{p}$  and  $3\mathbf{p}$                       **b**  $(\mathbf{p} + 2\mathbf{q})$  and  $(2\mathbf{p} - 4\mathbf{q})$       **c**  $(3\mathbf{p} - \mathbf{q})$  and  $(\mathbf{p} - \frac{1}{3}\mathbf{q})$

**d**  $(\mathbf{p} - 2\mathbf{q})$  and  $(4\mathbf{q} - 2\mathbf{p})$       **e**  $(\frac{3}{4}\mathbf{p} + \mathbf{q})$  and  $(6\mathbf{p} + 8\mathbf{q})$       **f**  $(2\mathbf{q} - 3\mathbf{p})$  and  $(\frac{3}{2}\mathbf{q} - \mathbf{p})$

**7** The points  $O, A, B$  and  $C$  are such that  $\overrightarrow{OA} = 4\mathbf{m}$ ,  $\overrightarrow{OB} = 4\mathbf{m} + 2\mathbf{n}$  and  $\overrightarrow{OC} = 2\mathbf{m} + 3\mathbf{n}$ , where  $\mathbf{m}$  and  $\mathbf{n}$  are non-parallel vectors.

**a** Find an expression for  $\overrightarrow{BC}$  in terms of  $\mathbf{m}$  and  $\mathbf{n}$ .

The point  $M$  is the mid-point of  $OC$ .

**b** Show that  $AM$  is parallel to  $BC$ .

**8** The points  $O, A, B$  and  $C$  are such that  $\overrightarrow{OA} = 6\mathbf{u} - 4\mathbf{v}$ ,  $\overrightarrow{OB} = 3\mathbf{u} - \mathbf{v}$  and  $\overrightarrow{OC} = \mathbf{v} - 3\mathbf{u}$ , where  $\mathbf{u}$  and  $\mathbf{v}$  are non-parallel vectors.

The point  $M$  is the mid-point of  $OA$  and the point  $N$  is the point on  $AB$  such that  $AN : NB = 1 : 2$

**a** Find  $\overrightarrow{OM}$  and  $\overrightarrow{ON}$ .

**b** Prove that  $C, M$  and  $N$  are collinear.

**9** Given that vectors  $\mathbf{p}$  and  $\mathbf{q}$  are not parallel, find the values of the constants  $a$  and  $b$  such that

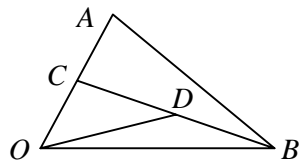
**a**  $a\mathbf{p} + 3\mathbf{q} = 5\mathbf{p} + b\mathbf{q}$

**b**  $(2\mathbf{p} + a\mathbf{q}) + (b\mathbf{p} - 4\mathbf{q}) = \mathbf{0}$

**c**  $4a\mathbf{q} - \mathbf{p} = b\mathbf{p} - 2\mathbf{q}$

**d**  $(2a\mathbf{p} + b\mathbf{q}) - (a\mathbf{q} - 6\mathbf{p}) = \mathbf{0}$

10



The diagram shows triangle  $OAB$  in which  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$ .

The point  $C$  is the mid-point of  $OA$  and the point  $D$  is the mid-point of  $BC$ .

**a** Find an expression for  $\overrightarrow{OD}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

**b** Show that if the point  $E$  lies on  $AB$  then  $\overrightarrow{OE}$  can be written in the form  $\mathbf{a} + k(\mathbf{b} - \mathbf{a})$ , where  $k$  is a constant.

Given also that  $OD$  produced meets  $AB$  at  $E$ ,

**c** find  $\overrightarrow{OE}$ ,

**d** show that  $AE : EB = 2 : 1$