C4 SERIES — Answers - Worksheet A

1 **a** $= 1 + (-1)x + \frac{(-1)(-2)}{2}x^2 + \frac{(-1)(-2)(-3)}{3\times 2}x^3 + \ldots$
$= 1 - x + x^2 - x^3 + \ldots$

b $= 1 + (\frac{1}{2})x + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}x^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3\times 2}x^3 + \ldots$
$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \ldots$

c $= 2[1 + (-3)x + \frac{(-3)(-4)}{2}x^2 + \frac{(-3)(-4)(-5)}{3\times 2}x^3 + \ldots]$
$= 2 - 6x + 12x^2 - 20x^3 + \ldots$

d $= 1 + (\frac{2}{3})x + \frac{(\frac{2}{3})(-\frac{1}{3})}{2}x^2 + \frac{(\frac{2}{3})(-\frac{1}{3})(-\frac{4}{3})}{3\times 2}x^3 + \ldots$
$= 1 + \frac{2}{3}x - \frac{1}{9}x^2 + \frac{4}{81}x^3 + \ldots$

e $= (1 - x)^{\frac{1}{3}} = 1 + (\frac{1}{3})(-x) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2}(-x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3\times 2}(-x)^3 + \ldots$
$= 1 - \frac{1}{3}x - \frac{1}{9}x^2 - \frac{5}{81}x^3 + \ldots$

f $= (1 + x)^{-2} = 1 + (-2)x + \frac{(-2)(-3)}{2}x^2 + \frac{(-2)(-3)(-4)}{3\times 2}x^3 + \ldots$
$= 1 - 2x + 3x^2 - 4x^3 + \ldots$

g $= \frac{1}{4}(1 - x)^{-4} = \frac{1}{4}[1 + (-4)(-x) + \frac{(-4)(-5)}{2}(-x)^2 + \frac{(-4)(-5)(-6)}{3\times 2}(-x)^3 + \ldots]$
$= \frac{1}{4} + x + \frac{5}{2}x^2 + 5x^3 + \ldots$

h $= 3(1 - x)^{-\frac{1}{2}} = 3[1 + (-\frac{1}{2})(-x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3\times 2}(-x)^3 + \ldots]$
$= 3 + \frac{3}{2}x + \frac{9}{8}x^2 + \frac{15}{16}x^3 + \ldots$

2 **a** $= 1 + (\frac{1}{2})(2x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(2x)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3\times 2}(2x)^3 + \ldots$
$= 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \ldots,\ |2x| < 1 \quad \therefore$ valid for $|x| < \frac{1}{2}$

b $= 1 + (-1)(-3x) + \frac{(-1)(-2)}{2}(-3x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-3x)^3 + \ldots$
$= 1 + 3x + 9x^2 + 27x^3 + \ldots,\ |-3x| < 1 \quad \therefore$ valid for $|x| < \frac{1}{3}$

c $= 1 + (-\frac{1}{2})(-4x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-4x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3\times 2}(-4x)^3 + \ldots$
$= 1 + 2x + 6x^2 + 20x^3 + \ldots,\ |-4x| < 1 \quad \therefore$ valid for $|x| < \frac{1}{4}$

d $= 1 + (-3)(\frac{1}{2}x) + \frac{(-3)(-4)}{2}(\frac{1}{2}x)^2 + \frac{(-3)(-4)(-5)}{3\times 2}(\frac{1}{2}x)^3 + \ldots$
$= 1 - \frac{3}{2}x + \frac{3}{2}x^2 - \frac{5}{4}x^3 + \ldots,\ |\frac{1}{2}x| < 1 \quad \therefore$ valid for $|x| < 2$

e $= 1 + (\frac{1}{3})(-6x) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2}(-6x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3\times 2}(-6x)^3 + \ldots$
$= 1 - 2x - 4x^2 - \frac{40}{3}x^3 + \ldots,\ |-6x| < 1 \quad \therefore$ valid for $|x| < \frac{1}{6}$

f $= 1 + (-4)(\frac{1}{4}x) + \frac{(-4)(-5)}{2}(\frac{1}{4}x)^2 + \frac{(-4)(-5)(-6)}{3\times 2}(\frac{1}{4}x)^3 + \ldots$
$= 1 - x + \frac{5}{8}x^2 - \frac{5}{16}x^3 + \ldots,\ |\frac{1}{4}x| < 1 \quad \therefore$ valid for $|x| < 4$

g $= 1 + (\frac{3}{2})(2x) + \frac{(\frac{3}{2})(\frac{1}{2})}{2}(2x)^2 + \frac{(\frac{3}{2})(\frac{1}{2})(-\frac{1}{2})}{3\times 2}(2x)^3 + \ldots$
$= 1 + 3x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \ldots,\ |2x| < 1 \quad \therefore$ valid for $|x| < \frac{1}{2}$

h $= 1 + (-\frac{4}{3})(-3x) + \frac{(-\frac{4}{3})(-\frac{7}{3})}{2}(-3x)^2 + \frac{(-\frac{4}{3})(-\frac{7}{3})(-\frac{10}{3})}{3\times 2}(-3x)^3 + \ldots$
$= 1 + 4x + 14x^2 + \frac{140}{3}x^3 + \ldots,\ |-3x| < 1 \quad \therefore$ valid for $|x| < \frac{1}{3}$

3 **a** $= 1 + (\frac{1}{2})(-2x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(-2x)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3\times 2}(-2x)^3 + \ldots$

$= 1 - x - \frac{1}{2}x^2 - \frac{1}{2}x^3 + \ldots$

b $\sqrt{0.98} = (1 - 2x)^{\frac{1}{2}}$ when $x = 0.01$

$\therefore \sqrt{0.98} \approx 1 - (0.01) - \frac{1}{2}(0.01)^2 - \frac{1}{2}(0.01)^3$

$= 1 - 0.01 - 0.000\,05 - 0.000\,000\,5$

$= 0.989\,949\,5$

c $\sqrt{0.98} = \sqrt{\frac{98}{100}} = \sqrt{\frac{49\times 2}{100}} = \frac{7}{10}\sqrt{2}$

$\therefore \sqrt{2} \approx \frac{10}{7} \times 0.989\,949\,5 = 1.414\,213\,6$ (8sf)

4 **a** $= 2^{-1}(1 + \frac{1}{2}x)^{-1} = \frac{1}{2}(1 + \frac{1}{2}x)^{-1}$

$= \frac{1}{2}[1 + (-1)(\frac{1}{2}x) + \frac{(-1)(-2)}{2}(\frac{1}{2}x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(\frac{1}{2}x)^3 + \ldots]$

$= \frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 + \ldots,\ |\frac{1}{2}x| < 1 \quad \therefore$ valid for $|x| < 2$

b $= 4^{\frac{1}{2}}(1+\frac{1}{4}x)^{\frac{1}{2}} = 2(1 + \frac{1}{4}x)^{\frac{1}{2}}$

$= 2[1 + (\frac{1}{2})(\frac{1}{4}x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(\frac{1}{4}x)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3\times 2}(\frac{1}{4}x)^3 + \ldots]$

$= 2 + \frac{1}{4}x - \frac{1}{64}x^2 + \frac{1}{512}x^3 + \ldots,\ |\frac{1}{4}x| < 1 \quad \therefore$ valid for $|x| < 4$

c $= 3^{-3}(1 - \frac{1}{3}x)^{-3} = \frac{1}{27}(1 - \frac{1}{3}x)^{-3}$

$= \frac{1}{27}[1 + (-3)(-\frac{1}{3}x) + \frac{(-3)(-4)}{2}(-\frac{1}{3}x)^2 + \frac{(-3)(-4)(-5)}{3\times 2}(-\frac{1}{3}x)^3 + \ldots]$

$= \frac{1}{27} + \frac{1}{27}x + \frac{2}{81}x^2 + \frac{10}{729}x^3 + \ldots,\ |-\frac{1}{3}x| < 1 \ \therefore$ valid for $|x| < 3$

d $= 9^{\frac{1}{2}}(1 + \frac{1}{3}x)^{\frac{1}{2}} = 3(1 + \frac{1}{3}x)^{\frac{1}{2}}$

$= 3[1 + (\frac{1}{2})(\frac{1}{3}x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(\frac{1}{3}x)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3\times 2}(\frac{1}{3}x)^3 + \ldots]$

$= 3 + \frac{1}{2}x - \frac{1}{24}x^2 + \frac{1}{144}x^3 + \ldots,\ |\frac{1}{3}x| < 1 \quad \therefore$ valid for $|x| < 3$

e $= 8^{\frac{1}{3}}(1 - 3x)^{\frac{1}{3}} = 2(1 - 3x)^{\frac{1}{3}}$

$= 2[1 + (\frac{1}{3})(-3x) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2}(-3x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3\times 2}(-3x)^3 + \ldots]$

$= 2 - 2x - 2x^2 - \frac{10}{3}x^3 + \ldots,\ |-3x| < 1 \quad \therefore$ valid for $|x| < \frac{1}{3}$

f $= 4^{-1}(1 - \frac{3}{4}x)^{-1} = \frac{1}{4}(1 - \frac{3}{4}x)^{-1}$

$= \frac{1}{4}[1 + (-1)(-\frac{3}{4}x) + \frac{(-1)(-2)}{2}(-\frac{3}{4}x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-\frac{3}{4}x)^3 + \ldots]$

$= \frac{1}{4} + \frac{3}{16}x + \frac{9}{64}x^2 + \frac{27}{256}x^3 + \ldots,\ |-\frac{3}{4}x| < 1 \ \therefore$ valid for $|x| < \frac{4}{3}$

g $= 4^{-\frac{1}{2}}(1 + \frac{3}{2}x)^{-\frac{1}{2}} = \frac{1}{2}(1 + \frac{3}{2}x)^{-\frac{1}{2}}$

$= \frac{1}{2}[1 + (-\frac{1}{2})(\frac{3}{2}x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(\frac{3}{2}x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3\times 2}(\frac{3}{2}x)^3 + \ldots]$

$= \frac{1}{2} - \frac{3}{8}x + \frac{27}{64}x^2 - \frac{135}{256}x^3 + \ldots,\ |\frac{3}{2}x| < 1 \quad \therefore$ valid for $|x| < \frac{2}{3}$

h $= 3^{-2}(1 + \frac{2}{3}x)^{-2} = \frac{1}{9}(1 + \frac{2}{3}x)^{-2}$

$= \frac{1}{9}[1 + (-2)(\frac{2}{3}x) + \frac{(-2)(-3)}{2}(\frac{2}{3}x)^2 + \frac{(-2)(-3)(-4)}{3\times 2}(\frac{2}{3}x)^3 + \ldots]$

$= \frac{1}{9} - \frac{4}{27}x + \frac{4}{27}x^2 - \frac{32}{243}x^3 + \ldots,\ |\frac{2}{3}x| < 1 \quad \therefore$ valid for $|x| < \frac{3}{2}$

5 **a** $= 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(2x)^3 + \ldots$

$= 1 - 2x + 4x^2 - 8x^3 + \ldots$

b $= (1-x)(1+2x)^{-1} = (1-x)(1 - 2x + 4x^2 - 8x^3 + \ldots)$

$= 1 - 2x + 4x^2 - 8x^3 - x + 2x^2 - 4x^3 + \ldots$

$= 1 - 3x + 6x^2 - 12x^3 + \ldots$

6 **a** $= (1+3x)(1-x)^{-1} = (1+3x)[1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-x)^3 + \ldots]$

$= (1+3x)(1 + x + x^2 + x^3 + \ldots)$

$= 1 + x + x^2 + x^3 + 3x + 3x^2 + 3x^3 + \ldots$

$= 1 + 4x + 4x^2 + 4x^3 + \ldots,\ |-x| < 1 \qquad \therefore$ valid for $|x| < 1$

b $= (2x-1)(1+4x)^{-2} = (2x-1)[1 + (-2)(4x) + \frac{(-2)(-3)}{2}(4x)^2 + \frac{(-2)(-3)(-4)}{3\times 2}(4x)^3 + \ldots]$

$= (2x-1)(1 - 8x + 48x^2 - 256x^3 + \ldots)$

$= 2x - 16x^2 + 96x^3 - 1 + 8x - 48x^2 + 256x^3 + \ldots$

$= -1 + 10x - 64x^2 + 352x^3 + \ldots,\ |4x| < 1 \quad \therefore$ valid for $|x| < \frac{1}{4}$

c $= (3+x)(2-x)^{-1} = (3+x) \times 2^{-1}(1 - \frac{1}{2}x)^{-1}$

$= (3+x) \times \frac{1}{2}[1 + (-1)(-\frac{1}{2}x) + \frac{(-1)(-2)}{2}(-\frac{1}{2}x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-\frac{1}{2}x)^3 + \ldots]$

$= (3+x)(\frac{1}{2} + \frac{1}{4}x + \frac{1}{8}x^2 + \frac{1}{16}x^3 + \ldots)$

$= \frac{3}{2} + \frac{3}{4}x + \frac{3}{8}x^2 + \frac{3}{16}x^3 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots$

$= \frac{3}{2} + \frac{5}{4}x + \frac{5}{8}x^2 + \frac{5}{16}x^3 + \ldots,\ |-\frac{1}{2}x| < 1 \qquad \therefore$ valid for $|x| < 2$

d $= (1-x)(1+2x)^{-\frac{1}{2}} = (1-x)[1 + (-\frac{1}{2})(2x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(2x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3\times 2}(2x)^3 + \ldots]$

$= (1-x)(1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3 + \ldots)$

$= 1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3 - x + x^2 - \frac{3}{2}x^3 + \ldots$

$= 1 - 2x + \frac{5}{2}x^2 - 4x^3 + \ldots,\ |2x| < 1 \qquad \therefore$ valid for $|x| < \frac{1}{2}$

7 **a** $\frac{x-2}{(1-x)(1-2x)} \equiv \frac{A}{1-x} + \frac{B}{1-2x}$

$x - 2 \equiv A(1-2x) + B(1-x)$

$x = 1 \quad \Rightarrow \quad -1 = -A \quad \Rightarrow \quad A = 1$

$x = \frac{1}{2} \quad \Rightarrow \quad -\frac{3}{2} = \frac{1}{2}B \quad \Rightarrow \quad B = -3$

$\therefore \frac{x-2}{(1-x)(1-2x)} \equiv \frac{1}{1-x} - \frac{3}{1-2x}$

b $\frac{1}{1-x} = (1-x)^{-1} = 1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-x)^3 + \ldots$

$= 1 + x + x^2 + x^3 + \ldots,\ |-x| < 1 \ \therefore |x| < 1$

$\frac{3}{1-2x} = 3(1-2x)^{-1} = 3[1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-2x)^3 + \ldots]$

$= 3 + 6x + 12x^2 + 24x^3 + \ldots,\ |-2x| < 1 \ \therefore |x| < \frac{1}{2}$

$\therefore \frac{x-2}{(1-x)(1-2x)} = (1 + x + x^2 + x^3 + \ldots) - (3 + 6x + 12x^2 + 24x^3 + \ldots)$

$= -2 - 5x - 11x^2 - 23x^3 + \ldots,$ valid for $|x| < \frac{1}{2}$

8 **a** $\dfrac{4}{(1+x)(1-3x)} \equiv \dfrac{A}{1+x} + \dfrac{B}{1-3x}$

$4 \equiv A(1-3x) + B(1+x)$

$x = -1 \quad \Rightarrow \quad 4 = 4A \quad \Rightarrow \quad A = 1$

$x = \frac{1}{3} \quad \Rightarrow \quad 4 = \frac{4}{3}B \quad \Rightarrow \quad B = 3$

$\therefore\ \mathrm{f}(x) \equiv \dfrac{1}{1+x} + \dfrac{3}{1-3x}$

$\dfrac{1}{1+x} = (1+x)^{-1} = 1 + (-1)x + \frac{(-1)(-2)}{2}x^2 + \frac{(-1)(-2)(-3)}{3\times 2}x^3 + \ldots$

$= 1 - x + x^2 - x^3 + \ldots, \ |x| < 1$

$\dfrac{3}{1-3x} = 3(1-3x)^{-1} = 3[1 + (-1)(-3x) + \frac{(-1)(-2)}{2}(-3x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-3x)^3 + \ldots]$

$= 3 + 9x + 27x^2 + 81x^3 + \ldots, \ |-3x| < 1 \ \ \therefore |x| < \frac{1}{3}$

$\therefore\ \mathrm{f}(x) \equiv (1 - x + x^2 - x^3 + \ldots) + (3 + 9x + 27x^2 + 81x^3 + \ldots)$

$\mathrm{f}(x) \equiv 4 + 8x + 28x^2 + 80x^3 + \ldots,$ valid for $|x| < \frac{1}{3}$

b $\dfrac{1-6x}{1+3x-4x^2} \equiv \dfrac{1-6x}{(1-x)(1+4x)} \equiv \dfrac{A}{1-x} + \dfrac{B}{1+4x}$

$1 - 6x \equiv A(1+4x) + B(1-x)$

$x = 1 \quad \Rightarrow \quad -5 = 5A \quad \Rightarrow \quad A = -1$

$x = -\frac{1}{4} \quad \Rightarrow \quad \frac{5}{2} = \frac{5}{4}B \quad \Rightarrow \quad B = 2$

$\therefore\ \mathrm{f}(x) \equiv \dfrac{2}{1+4x} - \dfrac{1}{1-x}$

$\dfrac{2}{1+4x} = 2(1+4x)^{-1} = 2[1 + (-1)(4x) + \frac{(-1)(-2)}{2}(4x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(4x)^3 + \ldots]$

$= 2 - 8x + 32x^2 - 128x^3 + \ldots, \ |4x| < 1 \ \ \therefore |x| < \frac{1}{4}$

$\dfrac{1}{1-x} = (1-x)^{-1} = 1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-x)^3 + \ldots$

$= 1 + x + x^2 + x^3 + \ldots, \ |-x| < 1 \ \ \therefore |x| < 1$

$\therefore\ \mathrm{f}(x) \equiv (2 - 8x + 32x^2 - 128x^3 + \ldots) - (1 + x + x^2 + x^3 + \ldots)$

$\mathrm{f}(x) \equiv 1 - 9x + 31x^2 - 129x^3 + \ldots,$ valid for $|x| < \frac{1}{4}$

c $\dfrac{5}{2-3x-2x^2} \equiv \dfrac{5}{(1-2x)(2+x)} = \dfrac{A}{1-2x} + \dfrac{B}{2+x}$

$5 \equiv A(2+x) + B(1-2x)$

$x = \frac{1}{2} \quad \Rightarrow \quad 5 = \frac{5}{2}A \quad \Rightarrow \quad A = 2$

$x = -2 \quad \Rightarrow \quad 5 = 5B \quad \Rightarrow \quad B = 1$

$\therefore\ \mathrm{f}(x) \equiv \dfrac{2}{1-2x} + \dfrac{1}{2+x}$

$\dfrac{2}{1-2x} = 2(1-2x)^{-1} = 2[1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-2x)^3 + \ldots]$

$= 2 + 4x + 8x^2 + 16x^3 + \ldots, \ |-2x| < 1 \ \ \therefore |x| < \frac{1}{2}$

$\dfrac{1}{2+x} = (2+x)^{-1} = 2^{-1}(1 + \frac{1}{2}x)^{-1} = \frac{1}{2}[1 + (-1)(\frac{1}{2}x) + \frac{(-1)(-2)}{2}(\frac{1}{2}x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(\frac{1}{2}x)^3 + \ldots]$

$= \frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 + \ldots, \ |\frac{1}{2}x| < 1 \ \ \therefore |x| < 2$

$\therefore\ \mathrm{f}(x) \equiv (2 + 4x + 8x^2 + 16x^3 + \ldots) + (\frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 + \ldots)$

$\mathrm{f}(x) \equiv \frac{5}{2} + \frac{15}{4}x + \frac{65}{8}x^2 + \frac{255}{16}x^3 + \ldots,$ valid for $|x| < \frac{1}{2}$

d $\dfrac{7x-3}{x^2-4x+3} \equiv \dfrac{7x-3}{(x-1)(x-3)} \equiv \dfrac{A}{x-1} + \dfrac{B}{x-3}$

$7x - 3 \equiv A(x - 3) + B(x - 1)$

$x = 1 \quad \Rightarrow \quad 4 = -2A \quad \Rightarrow \quad A = -2$

$x = 3 \quad \Rightarrow \quad 18 = 2B \quad \Rightarrow \quad B = 9$

$\therefore\ \mathrm{f}(x) \equiv \dfrac{9}{x-3} - \dfrac{2}{x-1} \equiv \dfrac{2}{1-x} - \dfrac{9}{3-x}$

$\dfrac{2}{1-x} = 2(1 - x)^{-1} = 2[1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-x)^3 + \ldots]$

$= 2 + 2x + 2x^2 + 2x^3 + \ldots,\ |-x| < 1\ \ \therefore |x| < 1$

$\dfrac{9}{3-x} = 9(3 - x)^{-1} = 9 \times 3^{-1}(1 - \frac{1}{3}x)^{-1}$

$= 3[1 + (-1)(-\frac{1}{3}x) + \frac{(-1)(-2)}{2}(-\frac{1}{3}x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-\frac{1}{3}x)^3 + \ldots]$

$= 3 + x + \frac{1}{3}x^2 + \frac{1}{9}x^3 + \ldots,\ |-\frac{1}{3}x| < 1\ \ \therefore |x| < 3$

$\therefore\ \mathrm{f}(x) \equiv (2 + 2x + 2x^2 + 2x^3 + \ldots) - (3 + x + \frac{1}{3}x^2 + \frac{1}{9}x^3 + \ldots)$

$\mathrm{f}(x) \equiv -1 + x + \frac{5}{3}x^2 + \frac{17}{9}x^3 + \ldots,$ valid for $|x| < 1$

e $\dfrac{3+5x}{(1+3x)(1+x)^2} \equiv \dfrac{A}{1+3x} + \dfrac{B}{1+x} + \dfrac{C}{(1+x)^2}$

$3 + 5x \equiv A(1 + x)^2 + B(1 + 3x)(1 + x) + C(1 + 3x)$

$x = -\frac{1}{3} \quad \Rightarrow \quad \frac{4}{3} = \frac{4}{9}A \quad \Rightarrow \quad A = 3$

$x = -1 \quad \Rightarrow \quad -2 = -2C \quad \Rightarrow \quad C = 1$

coeffs of x^2 $\Rightarrow \quad 0 = A + 3B \quad \Rightarrow \quad B = -1$

$\therefore\ \mathrm{f}(x) \equiv \dfrac{3}{1+3x} - \dfrac{1}{1+x} + \dfrac{1}{(1+x)^2}$

$\dfrac{3}{1+3x} = 3(1 + 3x)^{-1} = 3[1 + (-1)(3x) + \frac{(-1)(-2)}{2}(3x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(3x)^3 + \ldots]$

$= 3 - 9x + 27x^2 - 81x^3 + \ldots,\ |3x| < 1\ \ \therefore |x| < \frac{1}{3}$

$\dfrac{1}{1+x} = (1 + x)^{-1} = 1 + (-1)x + \frac{(-1)(-2)}{2}x^2 + \frac{(-1)(-2)(-3)}{3\times 2}x^3 + \ldots$

$= 1 - x + x^2 - x^3 + \ldots,\ |x| < 1$

$\dfrac{1}{(1+x)^2} = (1 + x)^{-2} = 1 + (-2)x + \frac{(-2)(-3)}{2}x^2 + \frac{(-2)(-3)(-4)}{3\times 2}x^3 + \ldots$

$= 1 - 2x + 3x^2 - 4x^3 + \ldots,\ |x| < 1$

$\therefore\ \mathrm{f}(x) \equiv (3 - 9x + 27x^2 - 81x^3 + \ldots) - (1 - x + x^2 - x^3 + \ldots) + (1 - 2x + 3x^2 - 4x^3 + \ldots)$

$\mathrm{f}(x) \equiv 3 - 10x + 29x^2 - 84x^3 + \ldots,$ valid for $|x| < \frac{1}{3}$

f

$$2x^2 + x - 1 \overline{)\,2x^2 + 0x + 4}$$
$$\underline{2x^2 + x - 1}$$
$$-x + 5$$

$\therefore \dfrac{2x^2+4}{2x^2+x-1} \equiv 1 + \dfrac{5-x}{2x^2+x-1}$

$\dfrac{5-x}{2x^2+x-1} \equiv \dfrac{5-x}{(2x-1)(x+1)} \equiv \dfrac{A}{2x-1} + \dfrac{B}{x+1}$

$5 - x \equiv A(x + 1) + B(2x - 1)$

$x = \frac{1}{2} \quad \Rightarrow \quad \frac{9}{2} = \frac{3}{2}A \quad \Rightarrow \quad A = 3$

$x = -1 \quad \Rightarrow \quad 6 = -3B \quad \Rightarrow \quad B = -2$

$\therefore\ \mathrm{f}(x) \equiv 1 + \dfrac{3}{2x-1} - \dfrac{2}{x+1} \equiv 1 - \dfrac{3}{1-2x} - \dfrac{2}{1+x}$

$\dfrac{3}{1-2x} = 3(1 - 2x)^{-1} = 3[1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(-2x)^3 + \ldots]$

$= 3 + 6x + 12x^2 + 24x^3 + \ldots, \ |-2x| < 1 \ \therefore |x| < \frac{1}{2}$

$\dfrac{2}{1+x} = 2(1 + x)^{-1} = 2[1 + (-1)x + \frac{(-1)(-2)}{2}x^2 + \frac{(-1)(-2)(-3)}{3\times2}x^3 + \ldots]$

$= 2 - 2x + 2x^2 - 2x^3 + \ldots, \ |x| < 1$

$\therefore\ \mathrm{f}(x) \equiv 1 - (3 + 6x + 12x^2 + 24x^3 + \ldots) - (2 - 2x + 2x^2 - 2x^3 + \ldots)$

$\mathrm{f}(x) \equiv -4 - 4x - 14x^2 - 22x^3 + \ldots,$ valid for $|x| < \frac{1}{2}$