

## C4 INTEGRATION

## Answers - Worksheet K

**1 a**  $y = \int (x+2)^3 dx$   
 $y = \frac{1}{4}(x+2)^4 + c$

**c**  $x = \int 3e^{2t} + 2 dt$   
 $x = \frac{3}{2}e^{2t} + 2t + c$

**e**  $N = \int t\sqrt{t^2+1} dt$   
 $N = \frac{1}{2} \int 2t(t^2+1)^{\frac{1}{2}} dt$   
 $N = \frac{1}{2} \times \frac{2}{3}(t^2+1)^{\frac{3}{2}} + c$   
 $N = \frac{1}{3}(t^2+1)^{\frac{3}{2}} + c$

**2 a**  $y = \int e^{-x} dx$   
 $y = -e^{-x} + c$   
 $y = 3$  when  $x = 0$   
 $\therefore 3 = -1 + c$   
 $c = 4$   
 $\therefore y = 4 - e^{-x}$

**c**  $\frac{du}{dx} = \frac{4x}{x^2-3}$   
 $u = \int \frac{4x}{x^2-3} dx = 2 \int \frac{2x}{x^2-3} dx$   
 $u = 2 \ln|x^2-3| + c$   
 $u = 5$  when  $x = 2$   
 $\therefore 5 = 0 + c$   
 $c = 5$   
 $\therefore u = 2 \ln|x^2-3| + 5$

**3 a**  $\frac{x-8}{x^2-x-6} \equiv \frac{A}{x-3} + \frac{B}{x+2}$   
 $x-8 \equiv A(x+2) + B(x-3)$   
 $x=3 \Rightarrow -5 = 5A \Rightarrow A = -1$   
 $x=-2 \Rightarrow -10 = -5B \Rightarrow B = 2$   
 $\frac{x-8}{x^2-x-6} \equiv \frac{2}{x+2} - \frac{1}{x-3}$

**b**  $y = \int 4 \cos 2x dx$   
 $y = 2 \sin 2x + c$

**d**  $\frac{dy}{dx} = \frac{1}{2-x}$   
 $y = \int \frac{1}{2-x} dx$   
 $y = -\ln|2-x| + c$

**f**  $y = \int xe^x dx$   
 $u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = e^x, v = e^x$   
 $y = xe^x - \int e^x dx$   
 $y = xe^x - e^x + c \quad [y = e^x(x-1) + c]$

**b**  $y = \int \tan^3 t \sec^2 t dt$   
 $y = \frac{1}{4} \tan^4 t + c$   
 $y = 1$  when  $t = \frac{\pi}{3}$   
 $\therefore 1 = \frac{1}{4}(\sqrt{3})^4 + c$   
 $c = 1 - \frac{9}{4} = -\frac{5}{4}$   
 $\therefore y = \frac{1}{4} \tan^4 t - \frac{5}{4} \quad [y = \frac{1}{4}(\tan^4 t - 5)]$

**d**  $y = \int 3 \cos^2 x dx$   
 $y = \frac{3}{2} \int (1 + \cos 2x) dx$   
 $y = \frac{3}{2}(x + \frac{1}{2} \sin 2x) + c = \frac{3}{4}(2x + \sin 2x) + c$   
 $y = \pi$  when  $x = \frac{\pi}{2}$   
 $\therefore \pi = \frac{3}{4}(\pi + 0) + c$   
 $c = \frac{\pi}{4}$   
 $\therefore y = \frac{3}{4}(2x + \sin 2x) + \frac{\pi}{4}$   
 $[y = \frac{1}{4}(6x + 3 \sin 2x + \pi)]$

**b**  $\frac{dy}{dx} = \frac{x-8}{x^2-x-6}$   
 $y = \int \frac{x-8}{x^2-x-6} dx = \int (\frac{2}{x+2} - \frac{1}{x-3}) dx$   
 $y = 2 \ln|x+2| - \ln|x-3| + c$   
 $y = \ln 9$  when  $x = 1$   
 $\therefore \ln 9 = 2 \ln 3 - \ln 2 + c$   
 $c = \ln 2 \quad (\ln 9 = \ln 3^2 = 2 \ln 3)$   
 $\therefore y = 2 \ln|x+2| - \ln|x-3| + \ln 2$   
 when  $x = 2, y = 2 \ln 4 - 0 + \ln 2 = \ln(4^2 \times 2)$   
 $= \ln 32$

$$4 \quad \mathbf{a} \quad \int \frac{1}{2y+3} dy = \int dx$$

$$\frac{1}{2} \ln |2y+3| = x + c$$

$$[ y = \frac{1}{2}(ke^{2x} - 3) ]$$

$$\mathbf{c} \quad \int \frac{1}{y} dy = \int x dx$$

$$\ln |y| = \frac{1}{2}x^2 + c$$

$$[ y = ke^{\frac{1}{2}x^2} ]$$

$$\mathbf{e} \quad \int y dy = \int (x^2 - 2) dx$$

$$\frac{1}{2}y^2 = \frac{1}{3}x^3 - 2x + c$$

$$[ y^2 = \frac{2}{3}x^3 - 4x + k ]$$

$$\mathbf{g} \quad \int e^{3-y} dy = \int x^{-\frac{1}{2}} dx$$

$$-e^{3-y} = 2x^{\frac{1}{2}} + c$$

$$[ y = 3 - \ln(k - 2\sqrt{x}) ]$$

$$\mathbf{i} \quad \int \frac{1}{y} dy = \int x \sin x dx$$

$$u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = \sin x, v = -\cos x$$

$$\ln |y| = -x \cos x + \int \cos x dx$$

$$\ln |y| = \sin x - x \cos x + c$$

$$[ y = ke^{\sin x - x \cos x} ]$$

$$\mathbf{k} \quad \int \frac{y-3}{y(y-1)} dy = \int x dx$$

$$\frac{y-3}{y(y-1)} \equiv \frac{A}{y} + \frac{B}{y-1}$$

$$y-3 \equiv A(y-1) + By$$

$$y=0 \Rightarrow A=3, y=1 \Rightarrow B=-2$$

$$\int \left( \frac{3}{y} - \frac{2}{y-1} \right) dy = \int x dx$$

$$3 \ln |y| - 2 \ln |y-1| = \frac{1}{2}x^2 + c$$

$$\mathbf{b} \quad \int \operatorname{cosec}^2 2y dy = \int dx$$

$$-\frac{1}{2} \cot 2y = x + c$$

$$[ \cot 2y = k - 2x ]$$

$$\mathbf{d} \quad \int \frac{1}{y} dy = \int \frac{1}{x+1} dx$$

$$\ln |y| = \ln |x+1| + c$$

$$[ y = k(x+1) ]$$

$$\mathbf{f} \quad \int \sec^2 y dy = \int 2 \cos x dx$$

$$\tan y = 2 \sin x + c$$

$$\mathbf{h} \quad y \frac{dy}{dx} = x(y^2 + 3)$$

$$\int \frac{y}{y^2+3} dy = \int x dx$$

$$\frac{1}{2} \int \frac{2y}{y^2+3} dy = \int x dx$$

$$\frac{1}{2} \ln |y^2+3| = \frac{1}{2}x^2 + c$$

$$[ y^2 = ke^{x^2} - 3 ]$$

$$\mathbf{j} \quad \frac{dy}{dx} = \frac{e^{2x}}{e^y}$$

$$\int e^y dy = \int e^{2x} dx$$

$$e^y = \frac{1}{2}e^{2x} + c$$

$$[ y = \ln \left( \frac{1}{2}e^{2x} + c \right) ]$$

$$\mathbf{l} \quad \int y^{-2} dy = \int \ln x dx$$

$$u = \ln x, \frac{du}{dx} = \frac{1}{x}; \frac{dv}{dx} = 1, v = x$$

$$-y^{-1} = x \ln x - \int dx$$

$$-y^{-1} = x \ln x - x + c$$

$$[ y = \frac{1}{x - x \ln x + k} ]$$

$$5 \quad \mathbf{a} \quad \int 2y \, dy = \int x \, dx$$

$$y^2 = \frac{1}{2}x^2 + c$$

$$y = 3 \text{ when } x = 4$$

$$\therefore 9 = 8 + c$$

$$c = 1$$

$$\therefore y^2 = \frac{1}{2}x^2 + 1$$

$$\mathbf{c} \quad \int \frac{1}{y} \, dy = \int \cot^2 x \, dx$$

$$\int \frac{1}{y} \, dy = \int (\operatorname{cosec}^2 x - 1) \, dx$$

$$\ln|y| = -\cot x - x + c$$

$$y = 1 \text{ when } x = \frac{\pi}{2}$$

$$\therefore 0 = 0 - \frac{\pi}{2} + c$$

$$c = \frac{\pi}{2}$$

$$\therefore \ln|y| = \frac{\pi}{2} - \cot x - x$$

$$\mathbf{e} \quad \int \cot y \, dy = \int x^2 \, dx$$

$$\int \frac{\cos y}{\sin y} \, dy = \int x^2 \, dx$$

$$\ln|\sin y| = \frac{1}{3}x^3 + c$$

$$y = \frac{\pi}{6} \text{ when } x = 0$$

$$\therefore \ln \frac{1}{2} = 0 + c$$

$$c = -\ln 2$$

$$\therefore \ln|\sin y| = \frac{1}{3}x^3 - \ln 2$$

$$[2 \sin y = e^{\frac{1}{3}x^3}]$$

$$\mathbf{g} \quad \int \sin y \, dy = \int xe^{-x} \, dx$$

$$u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = e^{-x}, v = -e^{-x}$$

$$-\cos y = -xe^{-x} + \int e^{-x} \, dx$$

$$-\cos y = -xe^{-x} - e^{-x} + c$$

$$\cos y = (x+1)e^{-x} + k$$

$$y = \pi \text{ when } x = -1$$

$$\therefore -1 = 0 + k$$

$$k = -1$$

$$\therefore \cos y = (x+1)e^{-x} - 1$$

$$\mathbf{b} \quad \int (y+1)^{-3} \, dy = \int dx$$

$$-\frac{1}{2}(y+1)^{-2} = x + c$$

$$(y+1)^{-2} = k - 2x$$

$$y = 0 \text{ when } x = 2$$

$$\therefore 1 = k - 4$$

$$k = 5$$

$$\therefore (y+1)^{-2} = 5 - 2x$$

$$[(y+1)^2 = \frac{1}{5-2x}]$$

$$\mathbf{d} \quad \int \frac{1}{y+2} \, dy = \int \frac{1}{x-1} \, dx$$

$$\ln|y+2| = \ln|x-1| + c$$

$$y = 6 \text{ when } x = 3$$

$$\therefore \ln 8 = \ln 2 + c$$

$$c = \ln 4$$

$$\therefore \ln|y+2| = \ln|x-1| + \ln 4$$

$$[y+2 = 4(x-1) \Rightarrow y = 4x - 6]$$

$$\mathbf{f} \quad \frac{dy}{dx} = \frac{\sqrt{y}}{\sqrt{x+3}}$$

$$\int y^{\frac{1}{2}} \, dy = \int (x+3)^{-\frac{1}{2}} \, dx$$

$$2y^{\frac{3}{2}} = 2(x+3)^{\frac{1}{2}} + c$$

$$\sqrt{y} = \sqrt{x+3} + k$$

$$y = 16 \text{ when } x = 1$$

$$\therefore 4 = 2 + k$$

$$k = 2$$

$$\therefore \sqrt{y} = \sqrt{x+3} + 2$$

$$[y = (\sqrt{x+3} + 2)^2]$$

$$\mathbf{h} \quad \int \frac{\sin y}{1+\cos y} \, dy = \int \frac{1}{2}x^{-2} \, dx$$

$$-\int \frac{-\sin y}{1+\cos y} \, dy = \int \frac{1}{2}x^{-2} \, dx$$

$$-\ln|1+\cos y| = -\frac{1}{2}x^{-1} + c$$

$$\ln|1+\cos y| = \frac{1}{2}x^{-1} + k$$

$$y = \frac{\pi}{3} \text{ when } x = 1$$

$$\therefore \ln \frac{3}{2} = \frac{1}{2} + k$$

$$k = \ln \frac{3}{2} - \frac{1}{2}$$

$$\therefore \ln|1+\cos y| = \frac{1}{2}x^{-1} + \ln \frac{3}{2} - \frac{1}{2}$$

$$[(1+\cos y)^2 = \frac{9}{4}e^{\frac{1-x}{x}}]$$