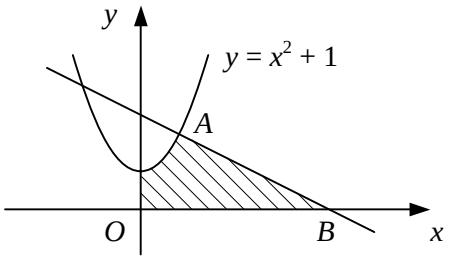
C4 INTEGRATION — Worksheet J

1



The diagram shows the curve $y = x^2 + 1$ which passes through the point A (1, 2).

a Find an equation of the normal to the curve at the point A.

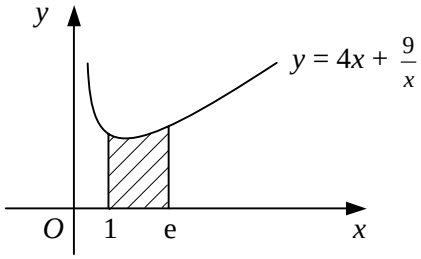
The normal to the curve at A meets the x-axis at the point B as shown.

b Find the coordinates of B.

The shaded region bounded by the curve, the coordinate axes and the line AB is rotated through 2π radians about the x-axis.

c Show that the volume of the solid formed is $\frac{36}{5}\pi$.

2



The shaded region in the diagram is bounded by the curve with equation $y = 4x + \frac{9}{x}$, the x-axis and the lines $x = 1$ and $x = \mathrm{e}$.

a Find the area of the shaded region, giving your answer in terms of e.

b Find, to 3 significant figures, the volume of the solid formed when the shaded region is rotated completely about the x-axis.

3 The region enclosed by the given curve, the x-axis and the given ordinates is rotated through 2π radians about the x-axis. Find the exact volume of the solid formed in each case.

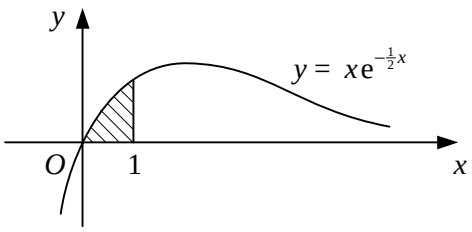
a $y = \operatorname{cosec} x$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{3}$

b $y = \sqrt{\frac{x+3}{x+2}}$, $x = 1$, $x = 4$

c $y = 1 + \cos 2x$, $x = 0$, $x = \frac{\pi}{4}$

d $y = x^{\frac{1}{2}}\mathrm{e}^{2-x}$, $x = 1$, $x = 2$

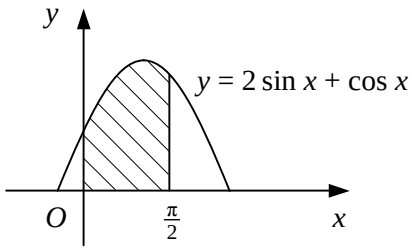
4



The shaded region in the diagram, bounded by the curve $y = x\mathrm{e}^{-\frac{1}{2}x}$, the x-axis and the line $x = 1$, is rotated through 360° about the x-axis.

Show that the volume of the solid formed is $\pi(2 - 5\mathrm{e}^{-1})$.

5



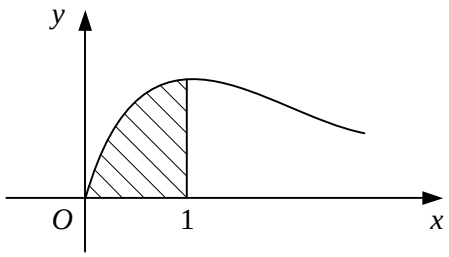
The diagram shows part of the curve with equation $y = 2\sin x + \cos x$.

The shaded region is bounded by the curve in the interval $0 \leq x < \frac{\pi}{2}$, the positive coordinate axes and the line $x = \frac{\pi}{2}$.

a Find the area of the shaded region.

b Show that the volume of the solid formed when the shaded region is rotated through 2π radians about the x-axis is $\frac{1}{4}\pi(5\pi + 8)$.

6



The diagram shows part of the curve with parametric equations

$$x = \tan\theta, \quad y = \sin 2\theta, \quad 0 \leq \theta < \frac{\pi}{2}.$$

The shaded region is bounded by the curve, the x-axis and the line $x = 1$.

a Write down the value of the parameter θ at the points where $x = 0$ and where $x = 1$.

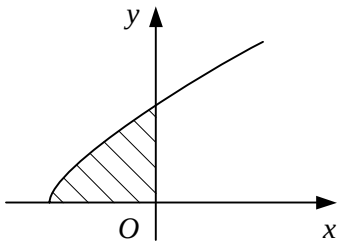
The shaded region is rotated through 2π radians about the x-axis.

b Show that the volume of the solid formed is given by

$$4\pi \int_0^{\frac{\pi}{4}} \sin^2\theta \ \mathrm{d}\theta.$$

c Evaluate this integral.

7



The diagram shows part of the curve with parametric equations

$$x = t^2 - 1, \quad y = t(t + 1), \quad t \geq 0.$$

a Find the value of the parameter t at the points where the curve meets the coordinate axes.

The shaded region bounded by the curve and the coordinate axes is rotated through 2π radians about the x-axis.

b Find the volume of the solid formed, giving your answer in terms of π.