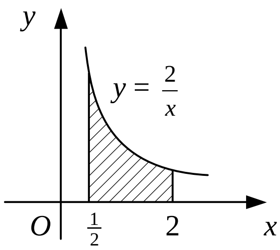
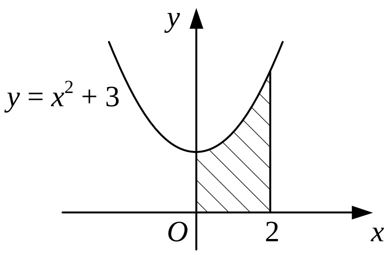
C4 INTEGRATION Worksheet I

1



The shaded region in the diagram is bounded by the curve $y = \frac{2}{x}$, the x-axis and the lines $x = \frac{1}{2}$ and $x = 2$. Show that when the shaded region is rotated through 360° about the x-axis, the volume of the solid formed is 6π.

2



The shaded region in the diagram, bounded by the curve $y = x^2 + 3$, the coordinate axes and the line $x = 2$, is rotated through 2π radians about the x-axis.

Show that the volume of the solid formed is approximately 127.

3 The region enclosed by the given curve, the x-axis and the given ordinates is rotated through 360° about the x-axis. Find the exact volume of the solid formed in each case.

a $y = 2e^{\frac{x}{2}}$, $x = 0$, $x = 1$

b $y = \frac{3}{x^2}$, $x = -2$, $x = -1$

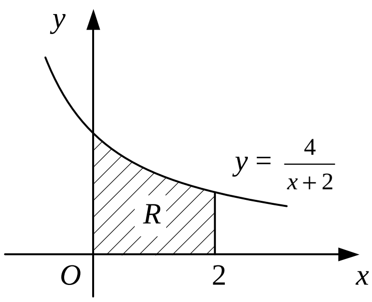
c $y = 1 + \frac{1}{x}$, $x = 3$, $x = 9$

d $y = \frac{3x^2 + 1}{x}$, $x = 1$, $x = 2$

e $y = \frac{1}{\sqrt{x+2}}$, $x = 2$, $x = 6$

f $y = e^{1-x}$, $x = -1$, $x = 1$

4



The diagram shows part of the curve with equation $y = \frac{4}{x+2}$.

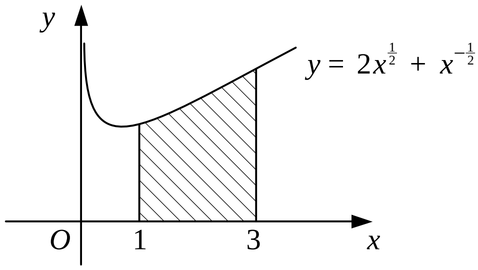
The shaded region, R, is bounded by the curve, the coordinate axes and the line $x = 2$.

a Find the area of R, giving your answer in the form $k \ln 2$.

The region R is rotated through 2π radians about the x-axis.

b Show that the volume of the solid formed is 4π.

5



The diagram shows the curve with equation $y = 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$.

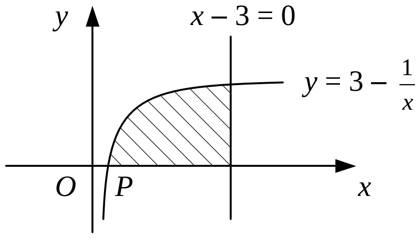
The shaded region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 3$ is rotated through 2π radians about the x-axis. Find the volume of the solid generated, giving your answer in the form $\pi(a + \ln b)$ where a and b are integers.

6 **a** Sketch the curve $y = 3x - x^2$, showing the coordinates of any points where the curve intersects the coordinate axes.

The region bounded by the curve and the x-axis is rotated through 360° about the x-axis.

b Show that the volume of the solid generated is $\frac{81}{10}\pi$.

7



The diagram shows the curve with equation $y = 3 - \frac{1}{x}$, $x > 0$.

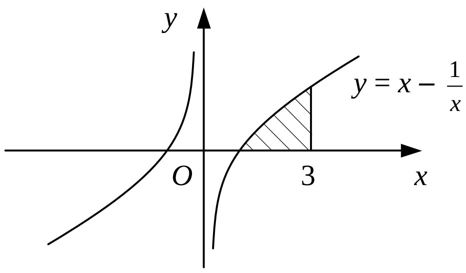
a Find the coordinates of the point P where the curve crosses the x-axis.

The shaded region is bounded by the curve, the straight line $x - 3 = 0$ and the x-axis.

b Find the area of the shaded region.

c Find the volume of the solid formed when the shaded region is rotated completely about the x-axis, giving your answer in the form $\pi(a + b \ln 3)$ where a and b are rational.

8



The diagram shows the curve $y = x - \frac{1}{x}$, $x \neq 0$.

a Find the coordinates of the points where the curve crosses the x-axis.

The shaded region is bounded by the curve, the x-axis and the line $x = 3$.

b Show that the area of the shaded region is $4 - \ln 3$.

The shaded region is rotated through 360° about the x-axis.

c Find the volume of the solid generated as an exact multiple of π.