

**C4** INTEGRATION

## Answers - Worksheet 1

$$\begin{aligned}
 1 \quad &= \pi \int_{\frac{1}{2}}^2 \left(\frac{2}{x}\right)^2 dx \\
 &= \pi \int_{\frac{1}{2}}^2 4x^{-2} dx \\
 &= \pi[-4x^{-1}]_{\frac{1}{2}}^2 \\
 &= \pi[-2 - (-8)] \\
 &= 6\pi
 \end{aligned}$$

$$\begin{aligned}
 2 \quad &= \pi \int_0^2 (x^2 + 3)^2 dx \\
 &= \pi \int_0^2 (x^4 + 6x^2 + 9) dx \\
 &= \pi \left[ \frac{1}{5}x^5 + 2x^3 + 9x \right]_0^2 \\
 &= \pi \left[ \left( \frac{32}{5} + 16 + 18 \right) - (0) \right] \\
 &= \frac{202}{5} \pi \approx 127
 \end{aligned}$$

$$\begin{aligned}
 3 \quad \mathbf{a} \quad &= \pi \int_0^1 (2e^{\frac{x}{2}})^2 dx \\
 &= \pi \int_0^1 4e^x dx \\
 &= \pi[4e^x]_0^1 \\
 &= \pi(4e - 4) \\
 &= 4\pi(e - 1)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad &= \pi \int_{-2}^{-1} \left(\frac{3}{x^2}\right)^2 dx \\
 &= \pi \int_{-2}^{-1} 9x^{-4} dx \\
 &= \pi[-3x^{-3}]_{-2}^{-1} \\
 &= \pi\left(3 - \frac{3}{8}\right) \\
 &= \frac{21}{8} \pi
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad &= \pi \int_3^9 \left(1 + \frac{1}{x}\right)^2 dx \\
 &= \pi \int_3^9 (1 + 2x^{-1} + x^{-2}) dx \\
 &= \pi \left[ x + 2 \ln|x| - x^{-1} \right]_3^9 \\
 &= \pi \left[ \left(9 + 2 \ln 9 - \frac{1}{9}\right) - \left(3 + 2 \ln 3 - \frac{1}{3}\right) \right] \\
 &= \pi \left(6\frac{2}{9} + 2 \ln 3\right)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad &= \pi \int_1^2 \left(3x + \frac{1}{x}\right)^2 dx \\
 &= \pi \int_1^2 (9x^2 + 6 + x^{-2}) dx \\
 &= \pi \left[ 3x^3 + 6x - x^{-1} \right]_1^2 \\
 &= \pi \left[ \left(24 + 12 - \frac{1}{2}\right) - \left(3 + 6 - 1\right) \right] \\
 &= \frac{55}{2} \pi
 \end{aligned}$$

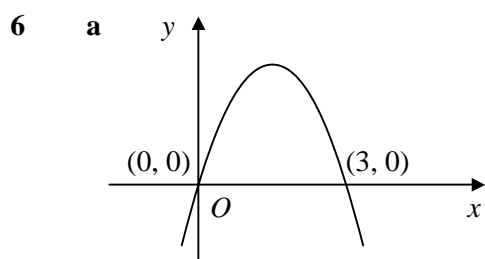
$$\begin{aligned}
 \mathbf{e} \quad &= \pi \int_2^6 \left(\frac{1}{\sqrt{x+2}}\right)^2 dx \\
 &= \pi \int_2^6 \frac{1}{x+2} dx \\
 &= \pi[\ln|x+2|]_2^6 \\
 &= \pi(\ln 8 - \ln 4) \\
 &= \pi \ln 2
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{f} \quad &= \pi \int_{-1}^1 (e^{1-x})^2 dx \\
 &= \pi \int_{-1}^1 e^{2-2x} dx \\
 &= \pi \left[ -\frac{1}{2} e^{2-2x} \right]_{-1}^1 \\
 &= -\frac{1}{2} \pi(1 - e^4) \\
 &= \frac{1}{2} \pi(e^4 - 1)
 \end{aligned}$$

$$\begin{aligned}
 4 \quad \mathbf{a} \quad &= \int_0^2 \frac{4}{x+2} dx \\
 &= [4 \ln|x+2|]_0^2 \\
 &= 4(\ln 4 - \ln 2) = 4 \ln 2
 \end{aligned}$$

$$\begin{aligned}
 5 \quad &= \pi \int_1^3 (2x^{\frac{1}{2}} + x^{-\frac{1}{2}})^2 dx \\
 &= \pi \int_1^3 (4x + 4 + x^{-1}) dx \\
 &= \pi [2x^2 + 4x + \ln|x|]_1^3 \\
 &= \pi [(18 + 12 + \ln 3) - (2 + 4 + 0)] \\
 &= \pi(24 + \ln 3)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad &= \pi \int_0^2 \left(\frac{4}{x+2}\right)^2 dx \\
 &= \pi \int_0^2 16(x+2)^{-2} dx \\
 &= \pi[-16(x+2)^{-1}]_0^2 \\
 &= \pi[-4 - (-8)] = 4\pi
 \end{aligned}$$



$$\begin{aligned}
 \mathbf{b} &= \pi \int_0^3 (3x - x^2)^2 dx \\
 &= \pi \int_0^3 (9x^2 - 6x^3 + x^4) dx \\
 &= \pi \left[ 3x^3 - \frac{3}{2}x^4 + \frac{1}{5}x^5 \right]_0^3 \\
 &= \pi \left[ (81 - \frac{243}{2} + \frac{243}{5}) - (0) \right] \\
 &= \frac{81}{10} \pi
 \end{aligned}$$

**8 a**  $x - \frac{1}{x} = 0$   
 $x^2 = 1$   
 $x = \pm 1 \therefore (-1, 0) \text{ and } (1, 0)$

$$\begin{aligned}
 \mathbf{b} &= \int_1^3 \left(x - \frac{1}{x}\right) dx \\
 &= \left[ \frac{1}{2}x^2 - \ln|x| \right]_1^3 \\
 &= \left( \frac{9}{2} - \ln 3 \right) - \left( \frac{1}{2} - 0 \right) \\
 &= 4 - \ln 3
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= \pi \int_1^3 \left(x - \frac{1}{x}\right)^2 dx \\
 &= \pi \int_1^3 (x^2 - 2 + x^{-2}) dx \\
 &= \pi \left[ \frac{1}{3}x^3 - 2x - x^{-1} \right]_1^3 \\
 &= \pi \left[ \left(9 - 6 - \frac{1}{3}\right) - \left(\frac{1}{3} - 2 - 1\right) \right] \\
 &= \frac{16}{3} \pi
 \end{aligned}$$

**7 a**  $y = 0 \therefore x = \frac{1}{3}$   
 $\therefore \left(\frac{1}{3}, 0\right)$

$$\begin{aligned}
 \mathbf{b} &= \int_{\frac{1}{3}}^3 \left(3 - \frac{1}{x}\right) dx \\
 &= \left[ 3x - \ln|x| \right]_{\frac{1}{3}}^3 \\
 &= (9 - \ln 3) - \left(1 - \ln \frac{1}{3}\right) \\
 &= 8 - 2 \ln 3
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= \pi \int_{\frac{1}{3}}^3 \left(3 - \frac{1}{x}\right)^2 dx \\
 &= \pi \int_{\frac{1}{3}}^3 (9 - 6x^{-1} + x^{-2}) dx \\
 &= \pi \left[ 9x - 6 \ln|x| - x^{-1} \right]_{\frac{1}{3}}^3 \\
 &= \pi \left[ \left(27 - 6 \ln 3 - \frac{1}{3}\right) - \left(3 - 6 \ln \frac{1}{3} - 3\right) \right] \\
 &= \pi \left( 26 \frac{2}{3} - 12 \ln 3 \right)
 \end{aligned}$$