C4 INTEGRATION — Worksheet H

1 Using an appropriate method, integrate with respect to x

a $(2x - 3)^4$ **b** $\text{cosec}^2 \frac{1}{2}x$ **c** $2e^{4x-1}$ **d** $\frac{2(x-1)}{x(x+1)}$

e $\frac{3}{\cos^2 2x}$ **f** $x(x^2 + 3)^3$ **g** $\sec^4 x \tan x$ **h** $\sqrt{7+2x}$

i xe^{3x} **j** $\frac{x+2}{x^2-2x-3}$ **k** $\frac{1}{4(x+1)^3}$ **l** $\tan^2 3x$

m $4\cos^2(2x + 1)$ **n** $\frac{3x}{1-x^2}$ **o** $x \sin 2x$ **p** $\frac{x+4}{x+2}$

2 Evaluate

a $\int_1^2 6e^{2x-3}\, dx$ **b** $\int_0^{\frac{\pi}{3}} \tan x\, dx$ **c** $\int_{-2}^{2} \frac{2}{x-3}\, dx$

d $\int_2^3 \frac{6+x}{4+3x-x^2}\, dx$ **e** $\int_1^2 (1-2x)^3\, dx$ **f** $\int_0^{\frac{\pi}{3}} \sin^2 x \sin 2x\, dx$

3 Using the given substitution, evaluate

a $\int_0^{\frac{3}{2}} \frac{1}{\sqrt{9-x^2}}\, dx$ $x = 3\sin u$ **b** $\int_0^1 x(1-3x)^3\, dx$ $u = 1 - 3x$

c $\int_2^{2\sqrt{3}} \frac{1}{4+x^2}\, dx$ $x = 2\tan u$ **d** $\int_{-1}^0 x^2\sqrt{x+1}\, dx$ $u^2 = x + 1$

4 Integrate with respect to x

a $\frac{2}{5-3x}$ **b** $(x+1)e^{x^2+2x}$ **c** $\frac{1-x}{2x+1}$ **d** $\sin 3x \cos 2x$

e $3x(x-1)^4$ **f** $\frac{3x^2+6x+2}{x^2+3x+2}$ **g** $\frac{5}{\sqrt[3]{2x-1}}$ **h** $\frac{\cos x}{2+3\sin x}$

i $\frac{x^2}{\sqrt{x^3-1}}$ **j** $(2 - \cot x)^2$ **k** $\frac{6x-5}{(x-1)(2x-1)^2}$ **l** x^2e^{-x}

5 Evaluate

a $\int_2^4 \frac{1}{3x-4}\, dx$ **b** $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{cosec}^2 x \cot^2 x\, dx$ **c** $\int_0^1 \frac{7-x^2}{(2-x)^2(3-x)}\, dx$

d $\int_0^{\frac{\pi}{2}} x\cos\frac{1}{2}x\, dx$ **e** $\int_1^5 \frac{1}{\sqrt{4x+5}}\, dx$ **f** $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 2\cos x \cos 3x\, dx$

g $\int_0^2 x\sqrt{2x^2+1}\, dx$ **h** $\int_0^1 \frac{x^2+1}{x-2}\, dx$ **i** $\int_0^1 (x-2)(x+1)^3\, dx$

6 Find the exact area of the region enclosed by the given curve, the x-axis and the given ordinates.

a $y = \frac{x}{(x^2+2)^3}$, $x = 1$, $x = 2$ **b** $y = \ln x$, $x = 2$, $x = 4$

7 Given that

$$\int_3^6 \frac{ax^2+b}{x}\, dx = 18 + 5\ln 2,$$

find the values of the rational constants a and b.

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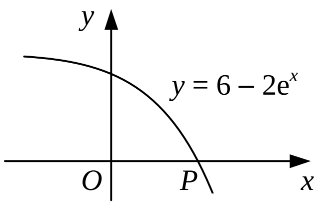

The diagram shows the curve with equation $y = 6 - 2e^x$.

a Find the coordinates of the point P where the curve crosses the x-axis.

b Show that the area of the region enclosed by the curve and the coordinate axes is $6 \ln 3 - 4$.

9 Using the substitution $u = \cot x$, show that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot^2 x \operatorname{cosec}^4 x \; dx = \tfrac{2}{15}(21\sqrt{3} - 4).$$

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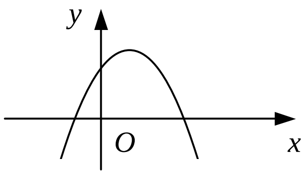

The diagram shows the curve with parametric equations

$$x = t + 1, \quad y = 4 - t^2.$$

a Show that the area of the region bounded by the curve and the x-axis is given by

$$\int_{-2}^{2} (4 - t^2) \; dt.$$

b Hence, find the area of this region.

11 **a** Given that k is a constant, show that

$$\frac{d}{dx}(x^2 \sin 2x + 2kx \cos 2x - k \sin 2x) = 2x^2 \cos 2x + (2 - 4k)x \sin 2x.$$

b Using your answer to part **a** with a suitable value of k, or otherwise, find

$$\int x^2 \cos 2x \; dx.$$

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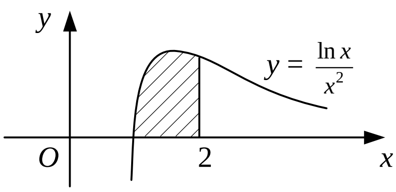

The shaded region in the diagram is bounded by the curve with equation $y = \frac{\ln x}{x^2}$, the x-axis and the line $x = 2$. Use integration by parts to show that the area of the shaded region is $\frac{1}{2}(1 - \ln 2)$.

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$$f(x) \equiv \frac{x + 16}{3x^3 + 11x^2 + 8x - 4}$$

a Factorise completely $3x^3 + 11x^2 + 8x - 4$.

b Express $f(x)$ in partial fractions.

c Show that $\int_{-1}^{0} f(x) \; dx = -(1 + 3 \ln 2)$.