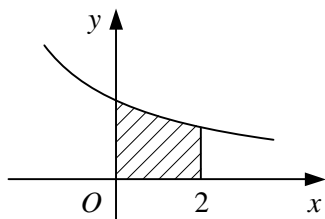


# C4 INTEGRATION

## Worksheet G

1



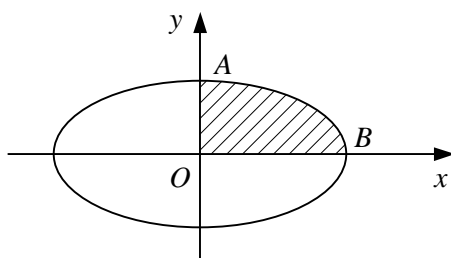
The diagram shows part of the curve with parametric equations

$$x = 2t - 4, \quad y = \frac{1}{t}.$$

The shaded region is bounded by the curve, the coordinate axes and the line  $x = 2$ .

- Find the value of the parameter  $t$  when  $x = 0$  and when  $x = 2$ .
- Show that the area of the shaded region is given by  $\int_2^3 \frac{2}{t} dt$ .
- Hence, find the area of the shaded region.
- Verify your answer to part c by first finding a cartesian equation for the curve.

2



The diagram shows the ellipse with parametric equations

$$x = 4 \cos \theta, \quad y = 2 \sin \theta, \quad 0 \leq \theta < 2\pi,$$

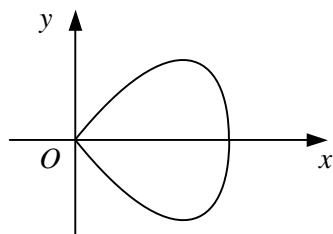
which meets the positive coordinate axes at the points  $A$  and  $B$ .

- Find the value of the parameter  $\theta$  at the points  $A$  and  $B$ .
- Show that the area of the shaded region bounded by the curve and the positive coordinate axes is given by

$$\int_0^{\frac{\pi}{2}} 8 \sin^2 \theta \, d\theta.$$

- Hence, show that the area of the region enclosed by the ellipse is  $8\pi$ .

3



The diagram shows the curve with parametric equations

$$x = 2 \sin t, \quad y = 5 \sin 2t, \quad 0 \leq t < \pi.$$

- Show that the area of the region enclosed by the curve is given by  $\int_0^{\frac{\pi}{2}} 20 \sin 2t \cos t \, dt$ .
- Evaluate this integral.