1 **a** $x = 0 \Rightarrow t = 2$

$x = 2 \Rightarrow t = 3$

b area $= \int_0^2 y \, dx$

$x = 2t - 4 \ \therefore \ \frac{dx}{dt} = 2$

$\therefore$ area $= \int_2^3 \frac{1}{t} \times 2 \, dt$

$= \int_2^3 \frac{2}{t} \, dt$

c $= [2 \ln |t|]_2^3$

$= 2 \ln 3 - 2 \ln 2$

$= 2 \ln \frac{3}{2}$

d $t = \frac{x+4}{2}$

$\therefore \ y = \frac{2}{x+4}$

$\therefore$ area $= \int_0^2 \frac{2}{x+4} \, dx$

$= [2 \ln |x + 4|]_0^2$

$= 2 \ln 6 - 2 \ln 4$

$= 2 \ln \frac{3}{2}$

2 **a** $x = 0 \Rightarrow \cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

for $y > 0, \ \theta = \frac{\pi}{2}$ at A

$y = 0 \Rightarrow \sin \theta = 0 \Rightarrow \theta = 0, \pi$

for $x > 0, \ \theta = 0$ at B

b $x = 4 \cos \theta \ \therefore \ \frac{dx}{d\theta} = -4 \sin \theta$

$\therefore$ area $= \int_{\frac{\pi}{2}}^0 2 \sin \theta \times -4 \sin \theta \, d\theta$

$= \int_0^{\frac{\pi}{2}} 8 \sin^2 \theta \, d\theta$

c shaded area $= \int_0^{\frac{\pi}{2}} (4 - 4 \cos 2\theta) \, d\theta$

$= [4\theta - 2 \sin 2\theta]_0^{\frac{\pi}{2}}$

$= (2\pi - 0) - (0 - 0)$

$= 2\pi$

area of ellipse $= 4 \times 2\pi$

$= 8\pi$

3 **a** $y = 0 \Rightarrow \sin 2t = 0 \Rightarrow t = 0, \frac{\pi}{2}$

$x = 2 \sin t \ \therefore \ \frac{dx}{dt} = 2 \cos t$

area above x-axis

$= \int_0^{\frac{\pi}{2}} 5 \sin 2t \times 2 \cos t \, dt$

$= \int_0^{\frac{\pi}{2}} 10 \sin 2t \cos t \, dt$

area enclosed by curve

$= 2 \int_0^{\frac{\pi}{2}} 10 \sin 2t \cos t \, dt$

$= \int_0^{\frac{\pi}{2}} 20 \sin 2t \cos t \, dt$

b $= 40 \int_0^{\frac{\pi}{2}} \sin t \cos^2 t \, dt$

$= -40 \int_0^{\frac{\pi}{2}} (-\sin t) \cos^2 t \, dt$

$= -40 [\frac{1}{3} \cos^3 t]_0^{\frac{\pi}{2}}$

$= -\frac{40}{3}(0 - 1)$

$= 13\frac{1}{3}$