

C4 INTEGRATION

Worksheet E

1 Showing your working in full, use the given substitution to find

a	$\int 2x(x^2 - 1)^3 dx$	$u = x^2 + 1$	b	$\int \sin^4 x \cos x dx$	$u = \sin x$
c	$\int 3x^2(2 + x^3)^2 dx$	$u = 2 + x^3$	d	$\int 2xe^{-x^2} dx$	$u = x^2$
e	$\int \frac{x}{(x^2 + 3)^4} dx$	$u = x^2 + 3$	f	$\int \sin 2x \cos^3 2x dx$	$u = \cos 2x$
g	$\int \frac{3x}{x^2 - 2} dx$	$u = x^2 - 2$	h	$\int x\sqrt{1 - x^2} dx$	$u = 1 - x^2$
i	$\int \sec^3 x \tan x dx$	$u = \sec x$	j	$\int (x + 1)(x^2 + 2x)^3 dx$	$u = x^2 + 2x$

2 **a** Given that $u = x^2 + 3$, find the value of u when

i $x = 0$

ii $x = 1$

b Using the substitution $u = x^2 + 3$, show that

$$\int_0^1 2x(x^2 + 3)^2 dx = \int_3^4 u^2 du.$$

c Hence, show that

$$\int_0^1 2x(x^2 + 3)^2 dx = 12\frac{1}{3}.$$

3 Using the given substitution, evaluate

a	$\int_1^2 x(x^2 - 3)^3 dx$	$u = x^2 - 3$	b	$\int_0^{\frac{\pi}{6}} \sin^3 x \cos x dx$	$u = \sin x$
c	$\int_0^3 \frac{4x}{x^2 + 1} dx$	$u = x^2 + 1$	d	$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x \sec^2 x dx$	$u = \tan x$
e	$\int_2^3 \frac{x}{\sqrt{x^2 - 3}} dx$	$u = x^2 - 3$	f	$\int_{-2}^{-1} x^2(x^3 + 2)^2 dx$	$u = x^3 + 2$
g	$\int_0^1 e^{2x}(1 + e^{2x})^3 dx$	$u = 1 + e^{2x}$	h	$\int_3^5 (x - 2)(x^2 - 4x)^2 dx$	$u = x^2 - 4x$

4 **a** Using the substitution $u = 4 - x^2$, show that

$$\int_0^2 x(4 - x^2)^3 dx = \int_0^4 \frac{1}{2}u^3 du.$$

b Hence, evaluate

$$\int_0^2 x(4 - x^2)^3 dx.$$

5 Using the given substitution, evaluate

a	$\int_0^1 xe^{2-x^2} dx$	$u = 2 - x^2$	b	$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos x} dx$	$u = 1 + \cos x$
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C4 INTEGRATION*Worksheet E continued*

- 6 a** By writing $\cot x$ as $\frac{\cos x}{\sin x}$, use the substitution $u = \sin x$ to show that

$$\int \cot x \, dx = \ln |\sin x| + c.$$

- b** Show that

$$\int \tan x \, dx = \ln |\sec x| + c.$$

- c** Evaluate

$$\int_0^{\frac{\pi}{6}} \tan 2x \, dx.$$

- 7** By recognising a function and its derivative, or by using a suitable substitution, integrate with respect to x

a $3x^2(x^3 - 2)^3$

b $e^{\sin x} \cos x$

c $\frac{x}{x^2 + 1}$

d $(2x + 3)(x^2 + 3x)^2$

e $x\sqrt{x^2 + 4}$

f $\cot^3 x \operatorname{cosec}^2 x$

g $\frac{e^x}{1 + e^x}$

h $\frac{\cos 2x}{3 + \sin 2x}$

i $\frac{x^3}{(x^4 - 2)^2}$

j $\frac{(\ln x)^3}{x}$

k $x^{\frac{1}{2}}(1 + x^{\frac{3}{2}})^2$

l $\frac{x}{\sqrt{5 - x^2}}$

- 8** Evaluate

a $\int_0^{\frac{\pi}{2}} \sin x (1 + \cos x)^2 \, dx$

b $\int_{-1}^0 \frac{e^{2x}}{2 - e^{2x}} \, dx$

c $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot x \operatorname{cosec}^4 x \, dx$

d $\int_2^4 \frac{x+1}{x^2 + 2x + 8} \, dx$

- 9** Using the substitution $u = x + 1$, show that

$$\int x(x+1)^3 \, dx = \frac{1}{20}(4x-1)(x+1)^4 + c.$$

- 10** Using the given substitution, find

a $\int x(2x-1)^4 \, dx$

$u = 2x - 1$

b $\int x\sqrt{1-x} \, dx$

$u^2 = 1 - x$

c $\int \frac{1}{(1-x^2)^{\frac{3}{2}}} \, dx$

$x = \sin u$

d $\int \frac{1}{\sqrt{x}-1} \, dx$

$x = u^2$

e $\int (x+1)(2x+3)^3 \, dx$

$u = 2x + 3$

f $\int \frac{x^2}{\sqrt{x-2}} \, dx$

$u^2 = x - 2$

- 11** Using the given substitution, evaluate

a $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} \, dx$

$x = \sin u$

b $\int_0^2 x(2-x)^3 \, dx$

$u = 2 - x$

c $\int_0^1 \sqrt{4-x^2} \, dx$

$x = 2 \sin u$

d $\int_0^3 \frac{x^2}{x^2+9} \, dx$

$x = 3 \tan u$