C4 INTEGRATION — *Answers - Worksheet D*

1 **a** $= 2\sin x + c$ **b** $= -\frac{1}{4}\cos 4x + c$ **c** $= 2\sin \frac{1}{2}x + c$ **d** $= -\cos(x + \frac{\pi}{4}) + c$

e $= \frac{1}{2}\sin(2x - 1) + c$ **f** $= 3\cos(\frac{\pi}{3} - x) + c$ **g** $= \sec x + c$ **h** $= -\cot x + c$

i $= \frac{5}{2}\tan 2x + c$ **j** $= -4\,\text{cosec}\,\frac{1}{4}x + c$ **k** $= \int 4\,\text{cosec}^2 x\ dx$ $= -4\cot x + c$ **l** $= \int \sec^2(4x + 1)\ dx$ $= \frac{1}{4}\tan(4x + 1) + c$

2 **a** $= [\sin x]_0^{\frac{\pi}{2}}$ $= 1 - 0 = 1$

b $= [-\frac{1}{2}\cos 2x]_0^{\frac{\pi}{6}}$ $= -\frac{1}{4} - (-\frac{1}{2}) = \frac{1}{4}$

c $= [4\sec \frac{1}{2}x]_0^{\frac{\pi}{2}}$ $= 4\sqrt{2} - 4 = 4(\sqrt{2} - 1)$

d $= [\frac{1}{2}\sin(2x - \frac{\pi}{3})]_0^{\frac{\pi}{3}}$ $= \frac{\sqrt{3}}{4} - (-\frac{\sqrt{3}}{4}) = \frac{\sqrt{3}}{2}$

e $= [\frac{1}{3}\tan 3x]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= 0 - (-\frac{1}{3}) = \frac{1}{3}$

f $= [-\text{cosec}\,x]_{\frac{\pi}{2}}^{\frac{2\pi}{3}}$ $= -\frac{2}{\sqrt{3}} - (-1) = 1 - \frac{2}{3}\sqrt{3}$

3 **a** $\tan^2\theta = \sec^2\theta - 1$

b $\int \tan^2 x\ dx = \int (\sec^2 x - 1)\ dx = \tan x - x + c$

4 **a** $\cos(A + B) \equiv \cos A\cos B - \sin A\sin B$

let $B = A \Rightarrow$
$\cos 2A \equiv \cos^2 A - \sin^2 A$
$\cos 2A \equiv \cos^2 A - (1 - \cos^2 A)$
$\cos 2A \equiv 2\cos^2 A - 1$
$\cos^2 A \equiv \frac{1}{2}(1 + \cos 2A)$

b $\int \cos^2 x\ dx = \int (\frac{1}{2} + \frac{1}{2}\cos 2x)\ dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$

5 **a** $= \int (\frac{1}{2} - \frac{1}{2}\cos 2x)\ dx$
$= \frac{1}{2}x - \frac{1}{4}\sin 2x + c$

b $= \int (\text{cosec}^2 2x - 1)\ dx$
$= -\frac{1}{2}\cot 2x - x + c$

c $= \int \frac{1}{2}\sin 2x\ dx$
$= -\frac{1}{4}\cos 2x + c$

d $= \int \frac{1}{\cos x} \times \frac{\sin x}{\cos x}\ dx$
$= \int \sec x\tan x\ dx$
$= \sec x + c$

e $= \int (2 + 2\cos 6x)\ dx$
$= 2x + \frac{1}{3}\sin 6x + c$

f $= \int (1 + 2\sin x + \sin^2 x)\ dx$
$= \int (1 + 2\sin x + \frac{1}{2} - \frac{1}{2}\cos 2x)\ dx$
$= \int (\frac{3}{2} + 2\sin x - \frac{1}{2}\cos 2x)\ dx$
$= \frac{3}{2}x - 2\cos x - \frac{1}{4}\sin 2x + c$

g $= \int (\sec^2 x - 2\sec x\tan x + \tan^2 x)\ dx$
$= \int (\sec^2 x - 2\sec x\tan x + \sec^2 x - 1)\ dx$
$= \int (2\sec^2 x - 2\sec x\tan x - 1)\ dx$
$= 2\tan x - 2\sec x - x + c$

h $= \int \frac{1}{\sin 2x} \times \frac{\cos x}{\sin x}\ dx$
$= \int \frac{1}{2\sin x\cos x} \times \frac{\cos x}{\sin x}\ dx$
$= \int \frac{1}{2}\text{cosec}^2 x\ dx$
$= -\frac{1}{2}\cot x + c$

i $= \int (\cos^2 x)^2 \, dx$

$= \int [\tfrac{1}{2}(1 + \cos 2x)]^2 \, dx$

$= \tfrac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x) \, dx$

$= \tfrac{1}{4}\int [1 + 2\cos 2x + \tfrac{1}{2}(1 + \cos 4x)] \, dx$

$= \tfrac{1}{8}\int (3 + 4\cos 2x + \cos 4x) \, dx$

$= \tfrac{1}{8}(3x + 2\sin 2x + \tfrac{1}{4}\sin 4x) + c$

$= \tfrac{3}{8}x + \tfrac{1}{4}\sin 2x + \tfrac{1}{32}\sin 4x + c$

6 **a** $= \int_0^{\frac{\pi}{2}} (1 + \cos 2x) \, dx$

$= [x + \tfrac{1}{2}\sin 2x]_0^{\frac{\pi}{2}}$

$= (\tfrac{\pi}{2} + 0) - (0 + 0) = \tfrac{\pi}{2}$

b $= \int_0^{\frac{\pi}{4}} \tfrac{1}{2}\sin 4x \, dx$

$= [-\tfrac{1}{8}\cos 4x]_0^{\frac{\pi}{4}}$

$= \tfrac{1}{8} - (-\tfrac{1}{8}) = \tfrac{1}{4}$

c $= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\sec^2 \tfrac{1}{2}x - 1) \, dx$

$= [2\tan \tfrac{1}{2}x - x]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$= (2 - \tfrac{\pi}{2}) - (\tfrac{2}{\sqrt{3}} - \tfrac{\pi}{3})$

$= 2 - \tfrac{2}{3}\sqrt{3} - \tfrac{\pi}{6}$

d $= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{\sin 2x} \times \frac{\cos 2x}{\sin 2x} \, dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} 2x \cot 2x \, dx$

$= [-\tfrac{1}{2}\operatorname{cosec} 2x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$= -\tfrac{1}{2} - (-\tfrac{1}{\sqrt{3}}) = \tfrac{1}{3}\sqrt{3} - \tfrac{1}{2}$

e $= \int_0^{\frac{\pi}{4}} (1 - 4\sin x + 4\sin^2 x) \, dx$

$= \int_0^{\frac{\pi}{4}} [1 - 4\sin x + 2(1 - \cos 2x)] \, dx$

$= \int_0^{\frac{\pi}{4}} (3 - 4\sin x - 2\cos 2x) \, dx$

$= [3x + 4\cos x - \sin 2x]_0^{\frac{\pi}{4}}$

$= (\tfrac{3\pi}{4} + 2\sqrt{2} - 1) - (0 + 4 - 0)$

$= \tfrac{3\pi}{4} + 2\sqrt{2} - 5$

f $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\cos^2 x} \times \frac{1}{\sin^2 x} \, dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{(\frac{1}{2}\sin 2x)^2} \, dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 4\operatorname{cosec}^2 2x \, dx$

$= [-2\cot 2x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= \tfrac{2}{\sqrt{3}} - (-\tfrac{2}{\sqrt{3}})$

$= \tfrac{4}{3}\sqrt{3}$

7 **a** $\sin(A + B) \equiv \sin A \cos B + \cos A \sin B$ (1)

$\sin(A - B) \equiv \sin A \cos B - \cos A \sin B$ (2)

(1) + (2) $\Rightarrow$ $\sin(A + B) + \sin(A - B) \equiv 2\sin A \cos B$

$\sin A \cos B \equiv \tfrac{1}{2}[\sin(A + B) + \sin(A - B)]$

b $= \int (\tfrac{1}{2}\sin 4x + \tfrac{1}{2}\sin 2x) \, dx = -\tfrac{1}{8}\cos 4x - \tfrac{1}{4}\cos 2x + c$

8 **a** $= \int (\cos 4x - \cos 6x) \, dx$

$= \tfrac{1}{4}\sin 4x - \tfrac{1}{6}\sin 6x + c$

b $= \int (\tfrac{1}{2}\cos 3x + \tfrac{1}{2}\cos x) \, dx$

$= \tfrac{1}{6}\sin 3x + \tfrac{1}{2}\sin x + c$

c $= \int [2\sin 5x + 2\sin(-3x)] \, dx$

$= \int (2\sin 5x - 2\sin 3x) \, dx$

$= -\tfrac{2}{5}\cos 5x + \tfrac{2}{3}\cos 3x + c$

d $= \int [\tfrac{1}{2}\sin(2x + \tfrac{\pi}{6}) - \tfrac{1}{2}\sin \tfrac{\pi}{6}] \, dx$

$= \int [\tfrac{1}{2}\sin(2x + \tfrac{\pi}{6}) - \tfrac{1}{4}] \, dx$

$= -\tfrac{1}{4}\cos(2x + \tfrac{\pi}{6}) - \tfrac{1}{4}x + c$