

C4 INTEGRATION

Worksheet C

1 a Express $\frac{3x+5}{(x+1)(x+3)}$ in partial fractions.

b Hence, find $\int \frac{3x+5}{(x+1)(x+3)} dx$.

2 Show that $\int \frac{3}{(t-2)(t+1)} dt = \ln \left| \frac{t-2}{t+1} \right| + c$.

3 Integrate with respect to x

a $\frac{6x-11}{(2x+1)(x-3)}$

b $\frac{14-x}{x^2+2x-8}$

c $\frac{6}{(2+x)(1-x)}$

d $\frac{x+1}{5x^2-14x+8}$

4 a Find the values of the constants A , B and C such that

$$\frac{x^2-6}{(x+4)(x-1)} \equiv A + \frac{B}{x+4} + \frac{C}{x-1}.$$

b Hence, find $\int \frac{x^2-6}{(x+4)(x-1)} dx$.

5 a Express $\frac{x^2-x-4}{(x+2)(x+1)^2}$ in partial fractions.

b Hence, find $\int \frac{x^2-x-4}{(x+2)(x+1)^2} dx$.

6 Integrate with respect to x

a $\frac{3x^2-5}{x^2-1}$

b $\frac{x(4x+13)}{(2+x)^2(3-x)}$

c $\frac{x^2-x+1}{x^2-3x-10}$

d $\frac{2-6x+5x^2}{x^2(1-2x)}$

7 Show that $\int_3^4 \frac{3x-5}{(x-1)(x-2)} dx = 2 \ln 3 - \ln 2$.

8 Find the exact value of

a $\int_1^3 \frac{x+3}{x(x+1)} dx$

b $\int_0^2 \frac{3x-2}{x^2+x-12} dx$

c $\int_1^2 \frac{9}{2x^2-7x-4} dx$

d $\int_0^2 \frac{2x^2-7x+7}{x^2-2x-3} dx$

e $\int_0^1 \frac{5x+7}{(x+1)^2(x+3)} dx$

f $\int_{-1}^1 \frac{2+x}{8-2x-x^2} dx$

9 a Express $\frac{1}{x^2-a^2}$, where a is a positive constant, in partial fractions.

b Hence, show that $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$.

c Find $\int \frac{1}{a^2-x^2} dx$.

10 Evaluate

a $\int_{-1}^1 \frac{1}{x^2-9} dx$

b $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{4}{1-x^2} dx$

c $\int_0^1 \frac{3}{2x^2-8} dx$