C4 INTEGRATION Worksheet A

1 Integrate with respect to x

a e^x **b** $4e^x$ **c** $\frac{1}{x}$ **d** $\frac{6}{x}$

2 Integrate with respect to t

a $2 + 3e^t$ **b** $t + t^{-1}$ **c** $t^2 - e^t$ **d** $9 - 2t^{-1}$

e $\frac{7}{t} + \sqrt{t}$ **f** $\frac{1}{4}e^t - \frac{1}{t}$ **g** $\frac{1}{3t} + \frac{1}{t^2}$ **h** $\frac{2}{5t} - \frac{3e^t}{7}$

3 Find

a $\int (5 - \frac{3}{x})\ dx$ **b** $\int (u^{-1} + u^{-2})\ du$ **c** $\int \frac{2e^t + 1}{5}\ dt$

d $\int \frac{3y+1}{y}\ dy$ **e** $\int (\frac{3}{4}e^t + 3\sqrt{t}\ )\ dt$ **f** $\int (x - \frac{1}{x})^2\ dx$

4 The curve $y = f(x)$ passes through the point $(1, -3)$.

Given that $f'(x) = \frac{(2x-1)^2}{x}$, find an expression for $f(x)$.

5 Evaluate

a $\int_0^1 (e^x + 10)\ dx$ **b** $\int_2^5 (t + \frac{1}{t})\ dt$ **c** $\int_1^4 \frac{5 - x^2}{x}\ dx$

d $\int_{-2}^{-1} \frac{6y+1}{3y}\ dy$ **e** $\int_{-3}^{3} (e^x - x^2)\ dx$ **f** $\int_2^3 \frac{4r^2 - 3r + 6}{r^2}\ dr$

g $\int_{\ln 2}^{\ln 4} (7 - e^u)\ du$ **h** $\int_6^{10} r^{-\frac{1}{2}}(2r^{\frac{1}{2}} + 9r^{-\frac{1}{2}})\ dr$ **i** $\int_4^9 (\frac{1}{\sqrt{x}} + 3e^x)\ dx$

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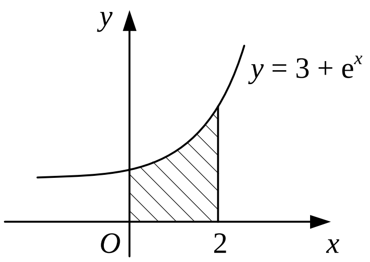

The shaded region on the diagram is bounded by the curve $y = 3 + e^x$, the coordinate axes and the line $x = 2$. Show that the area of the shaded region is $e^2 + 5$.

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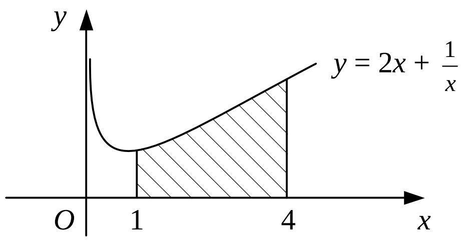

The shaded region on the diagram is bounded by the curve $y = 2x + \frac{1}{x}$, the x-axis and the lines $x = 1$ and $x = 4$. Find the area of the shaded region in the form $a + b \ln 2$.

8 Find the exact area of the region enclosed by the given curve, the x-axis and the given ordinates. In each case, $y > 0$ over the interval being considered.

a $y = 4x + 2e^x$, $x = 0$, $x = 1$

b $y = 1 + \frac{3}{x}$, $x = 2$, $x = 4$

c $y = 4 - \frac{1}{x}$, $x = -3$, $x = -1$

d $y = 2 - \frac{1}{2}e^x$, $x = 0$, $x = \ln 2$

e $y = e^x + \frac{5}{x}$, $x = \frac{1}{2}$, $x = 2$

f $y = \frac{x^3 - 2}{x}$, $x = 2$, $x = 3$

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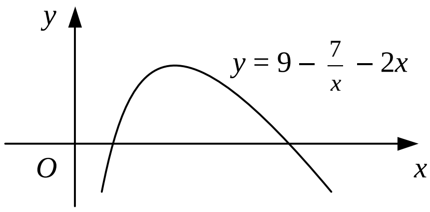

The diagram shows the curve with equation $y = 9 - \frac{7}{x} - 2x$, $x > 0$.

a Find the coordinates of the points where the curve crosses the x-axis.

b Show that the area of the region bounded by the curve and the x-axis is $11\frac{1}{4} - 7\ln\frac{7}{2}$.

10 **a** Sketch the curve $y = e^x - a$ where a is a constant and $a > 1$.

Show on your sketch the coordinates of any points of intersection with the coordinate axes and the equation of any asymptotes.

b Find, in terms of a, the area of the finite region bounded by the curve $y = e^x - a$ and the coordinate axes.

c Given that the area of this region is $1 + a$, show that $a = e^2$.

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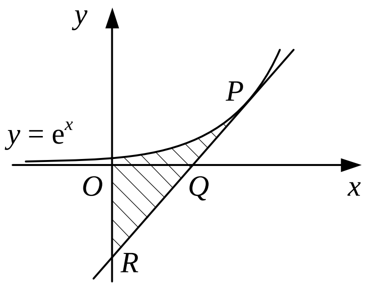

The diagram shows the curve with equation $y = e^x$. The point P on the curve has x-coordinate 3, and the tangent to the curve at P intersects the x-axis at Q and the y-axis at R.

a Find an equation of the tangent to the curve at P.

b Find the coordinates of the points Q and R.

The shaded region is bounded by the curve, the tangent to the curve at P and the y-axis.

c Find the exact area of the shaded region.

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$$f(x) \equiv \left(\frac{3}{\sqrt{x}} - 4\right)^2, \quad x \in \mathbb{R}, \quad x > 0.$$

a Find the coordinates of the point where the curve $y = f(x)$ meets the x-axis.

The finite region R is bounded by the curve $y = f(x)$, the line $x = 1$ and the x-axis.

b Show that the area of R is approximately 0.178