C4 DIFFERENTIATION Worksheet F

1 A curve has parametric equations

$$x = t^2, \quad y = \frac{2}{t}.$$

a Find $\frac{dy}{dx}$ in terms of t. **(3)**

b Find an equation for the normal to the curve at the point where $t = 2$, giving your answer in the form $y = mx + c$. **(3)**

2 A curve has the equation $y = 4^x$.

Show that the tangent to the curve at the point where $x = 1$ has the equation

$$y = 4 + 8(x - 1) \ln 2.$$ **(4)**

3 A curve has parametric equations

$$x = \sec \theta, \quad y = \cos 2\theta, \quad 0 \le \theta < \tfrac{\pi}{2}.$$

a Show that $\frac{dy}{dx} = -4 \cos^3 \theta$. **(4)**

b Show that the tangent to the curve at the point where $\theta = \frac{\pi}{6}$ has the equation

$$3\sqrt{3}\,x + 2y = k,$$

where k is an integer to be found. **(4)**

4 A curve has the equation

$$2x^2 + 6xy - y^2 + 77 = 0$$

and passes through the point P (2, –5).

a Show that the normal to the curve at P has the equation

$$x + y + 3 = 0.$$ **(6)**

b Find the x-coordinate of the point where the normal to the curve at P intersects the curve again. **(3)**

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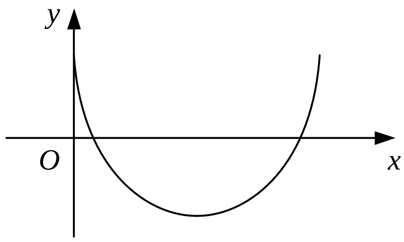

The diagram shows the curve with parametric equations

$$x = \theta - \sin \theta, \quad y = \cos \theta, \quad 0 \le \theta \le 2\pi.$$

a Find the exact coordinates of the points where the curve crosses the x-axis. **(3)**

b Show that $\frac{dy}{dx} = -\cot \frac{\theta}{2}$. **(5)**

c Find the exact coordinates of the point on the curve where the tangent to the curve is parallel to the x-axis. **(2)**

6 A curve has parametric equations

$$x = \sin\theta,\quad y = \sec^2\theta,\quad -\tfrac{\pi}{2} < \theta < \tfrac{\pi}{2}.$$

The point P on the curve has x-coordinate $\tfrac{1}{2}$.

a Write down the value of the parameter θ at P. **(1)**

b Show that the tangent to the curve at P has the equation

$$16x - 9y + 4 = 0.$$ **(6)**

c Find a cartesian equation for the curve. **(2)**

7 A curve has the equation

$$2\sin x - \tan 2y = 0.$$

a Show that $\dfrac{dy}{dx} = \cos x \cos^2 2y$. **(4)**

b Find an equation for the tangent to the curve at the point $(\tfrac{\pi}{3}, \tfrac{\pi}{6})$, giving your answer in the form $ax + by + c = 0$. **(3)**

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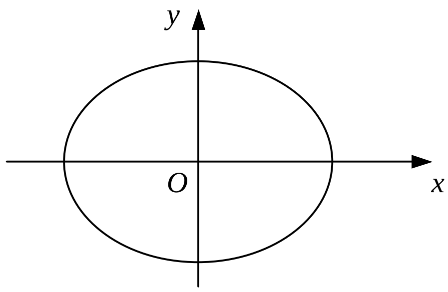

A particle moves on the ellipse shown in the diagram such that at time t its coordinates are given by

$$x = 4\cos t,\quad y = 3\sin t,\quad t \ge 0.$$

a Find $\dfrac{dy}{dx}$ in terms of t. **(3)**

b Show that at time t, the tangent to the path of the particle has the equation

$$3x\cos t + 4y\sin t = 12.$$ **(3)**

c Find a cartesian equation for the path of the particle. **(3)**

9 The curve with parametric equations

$$x = \frac{t}{t+1},\quad y = \frac{2t}{t-1},$$

passes through the origin, O.

a Show that $\dfrac{dy}{dx} = -2\left(\dfrac{t+1}{t-1}\right)^2$. **(4)**

b Find an equation for the normal to the curve at O. **(2)**

c Find the coordinates of the point where the normal to the curve at O meets the curve again. **(4)**

d Show that the cartesian equation of the curve can be written in the form

$$y = \frac{2x}{2x-1}.$$ **(4)**