

C4 DIFFERENTIATION

Worksheet E

- 1 A curve has the equation

$$3x^2 + xy - y^2 + 9 = 0.$$

Find an expression for $\frac{dy}{dx}$ in terms of x and y . (5)

- 2 A curve has parametric equations

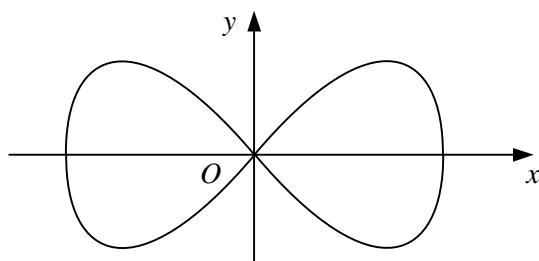
$$x = a \cos \theta, \quad y = a(\sin \theta - \theta), \quad 0 \leq \theta < \pi,$$

where a is a positive constant.

a Show that $\frac{dy}{dx} = \tan \frac{\theta}{2}$. (5)

b Find, in terms of a , an equation for the tangent to the curve at the point where it crosses the y -axis. (3)

- 3



The diagram shows the curve with parametric equations

$$x = \cos \theta, \quad y = \frac{1}{2} \sin 2\theta, \quad 0 \leq \theta < 2\pi.$$

a Find $\frac{dy}{dx}$ in terms of θ . (3)

b Find the two values of θ for which the curve passes through the origin. (2)

c Show that the two tangents to the curve at the origin are perpendicular to each other. (2)

d Find a cartesian equation for the curve. (4)

- 4 A curve has the equation

$$x^2 - 4xy + y^2 = 24.$$

a Show that $\frac{dy}{dx} = \frac{x-2y}{2x-y}$. (4)

b Find an equation for the tangent to the curve at the point $P(2, 10)$. (3)

The tangent to the curve at Q is parallel to the tangent at P .

c Find the coordinates of Q . (4)

- 5 A curve is given by the parametric equations

$$x = t^2 + 2, \quad y = t(t - 1).$$

a Find the coordinates of any points on the curve where the tangent to the curve is parallel to the x -axis. (5)

b Show that the tangent to the curve at the point $(3, 2)$ has the equation

$$3x - 2y = 5. \quad (5)$$

C4 DIFFERENTIATION

Worksheet E continued

- 6 Find an equation for the normal to the curve with equation

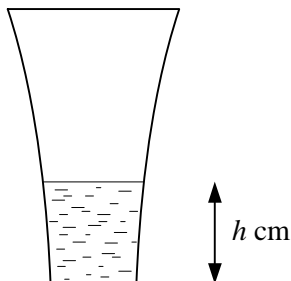
$$x^3 - 3x + xy - 2y^2 + 3 = 0$$

at the point (1, 1).

Give your answer in the form $y = mx + c$.

(7)

7



The diagram shows the cross-section of a vase. The volume of water in the vase, $V \text{ cm}^3$, when the depth of water in the vase is $h \text{ cm}$ is given by

$$V = 40\pi(e^{0.1h} - 1).$$

The vase is initially empty and water is poured into it at a constant rate of $80 \text{ cm}^3 \text{ s}^{-1}$.

Find the rate at which the depth of water in the vase is increasing

a when $h = 4$,

(5)

b after 5 seconds of pouring water in.

(4)

- 8 A curve is given by the parametric equations

$$x = \frac{t}{1+t}, \quad y = \frac{t}{1-t}, \quad t \neq \pm 1.$$

a Show that $\frac{dy}{dx} = \left(\frac{1+t}{1-t}\right)^2$.

(4)

b Show that the normal to the curve at the point P , where $t = \frac{1}{2}$, has the equation

$$3x + 27y = 28.$$

(4)

The normal to the curve at P meets the curve again at the point Q .

c Find the exact value of the parameter t at Q .

(4)

- 9 A curve has the equation

$$2x + x^2y - y^2 = 0.$$

Find the coordinates of the point on the curve where the tangent is parallel to the x -axis.

(8)

- 10 A curve has parametric equations

$$x = a \sec \theta, \quad y = 2a \tan \theta, \quad -\frac{\pi}{2} \leq \theta < \frac{\pi}{2},$$

where a is a positive constant.

a Find $\frac{dy}{dx}$ in terms of θ .

(3)

b Show that the normal to the curve at the point where $\theta = \frac{\pi}{4}$ has the equation

$$x + 2\sqrt{2}y = 5\sqrt{2}a.$$

(4)

c Find a cartesian equation for the curve in the form $y^2 = f(x)$.

(3)