

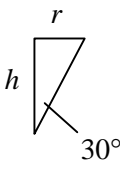
C4 DIFFERENTIATION

Answers - Worksheet D

- 1 a i** $\frac{dy}{dx} = 2x + 3$
- ii** $\frac{dx}{dt} = 3(t - 4)^2$
- b i** $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$
 $t = 5, \frac{dx}{dt} = 3, x = 1, \frac{dy}{dx} = 5$
 $\therefore \frac{dy}{dt} = 5 \times 3 = 15$
- ii** $x = 8, \frac{dy}{dx} = 19$
 $8 = (t - 4)^3 \therefore t = 6, \frac{dx}{dt} = 12$
 $\therefore \frac{dy}{dt} = 19 \times 12 = 228$
- 2** $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$
 $\frac{dy}{dx} = 1 \times \sqrt{2x-3} + x \times \frac{1}{2}(2x-3)^{-\frac{1}{2}} \times 2$
 $= \frac{(2x-3)+x}{\sqrt{2x-3}} = \frac{3(x-1)}{\sqrt{2x-3}}$
 $x = 6, \frac{dx}{dt} = 0.3, \frac{dy}{dx} = 5$
 $\therefore \frac{dy}{dt} = 5 \times 0.3 = 1.5$
 $\therefore y$ increasing at 1.5 units per second
- 3 a** $\frac{dP}{dt} = \frac{dP}{dr} \times \frac{dr}{dt}$
 $\frac{dr}{dt} = 0.2, P = 2\pi r \therefore \frac{dP}{dr} = 2\pi$
 $\therefore \frac{dP}{dt} = 2\pi \times 0.2 = 0.4\pi$
 \therefore perimeter increasing at $0.4\pi \text{ cm s}^{-1}$
- b** $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$
 $A = \pi r^2 \therefore \frac{dA}{dr} = 2\pi r$
 $r = 10, \frac{dA}{dr} = 20\pi$
 $\therefore \frac{dA}{dt} = 20\pi \times 0.2 = 4\pi \text{ cm}^2 \text{ s}^{-1}$
- c** $20 = 2\pi r \times 0.2 \therefore r = \frac{50}{\pi} = 15.9 \text{ cm (3sf)}$
- 4 a** $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$
 $A = \pi r^2 \therefore \frac{dA}{dr} = 2\pi r$
 $\frac{dA}{dt} = -0.5, r = 8 \therefore \frac{dA}{dr} = 16\pi$
 $\therefore -0.5 = 16\pi \times \frac{dr}{dt}$
 $\frac{dr}{dt} = -\frac{1}{32\pi} = -0.00995 \text{ cm s}^{-1}$
 \therefore radius decreasing at $0.00995 \text{ cm s}^{-1}$ (3sf)
- b** $\frac{dP}{dt} = \frac{dP}{dr} \times \frac{dr}{dt}$
 $P = 2\pi r \therefore \frac{dP}{dr} = 2\pi$
 $\therefore \frac{dP}{dt} = 2\pi \times -\frac{1}{32\pi} = -\frac{1}{16}$
 \therefore perimeter decreasing at 0.0625 cm s^{-1}

5 a $\frac{dV}{dt} = \frac{dV}{dl} \times \frac{dl}{dt}$
 $\frac{dV}{dt} = 3.5, V = l^3 \therefore \frac{dV}{dl} = 3l^2$
 $200 = l^3 \therefore l = \sqrt[3]{200} = 5.848$
 $\therefore \frac{dV}{dt} = 3 \times (5.848)^2 = 102.6$
 $\therefore 3.5 = 102.6 \times \frac{dl}{dt}$
 $\frac{dl}{dt} = 3.5 \div 102.6 = 0.0341 \text{ cm s}^{-1}$

b $2 \text{ mm s}^{-1} = 0.2 \text{ cm s}^{-1}$
 $3.5 = \frac{dV}{dt} \times 0.2$
 $\therefore \frac{dV}{dt} = 3.5 \div 0.2 = 17.5$
 $\therefore 17.5 = 3l^2$
 $l = \sqrt{\frac{17.5}{3}} = 2.415$
 $V = (2.415)^3 = 14.1 \text{ cm}^3 \text{ (3sf)}$

6 a  $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{r}{h}$
 $\therefore r = \frac{h}{\sqrt{3}}$
 $V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times \frac{1}{3} h^2 \times h = \frac{1}{9} \pi h^3$

b $t = 120, V = 565.06$
 $\therefore h = \sqrt[3]{\frac{9 \times 565.06}{\pi}} = 11.7 \text{ cm (3sf)}$

c $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$
 $\frac{dV}{dh} = \frac{1}{3} \pi h^2$
 $h = 11.74, \frac{dV}{dh} = 144.4$
 $\frac{dV}{dt} = 600 \times (-0.0005) e^{-0.0005t}$
 $= -0.3 e^{-0.0005t}$
 $t = 120, \frac{dV}{dt} = -0.2825$
 $\therefore -0.2825 = 144.4 \times \frac{dh}{dt}$
 $\frac{dh}{dt} = -0.00196$
 $\therefore \text{depth decreasing at } 0.00196 \text{ cm s}^{-1} \text{ (3sf)}$