

C4 DIFFERENTIATION

Answers - Worksheet C

- 1** **a** $= 4 \frac{dy}{dx}$ **b** $= 3y^2 \frac{dy}{dx}$ **c** $= 2 \frac{dy}{dx} \cos 2y$ **d** $= 3e^{y^2} \times 2y \frac{dy}{dx}$
 $= 6ye^{y^2} \frac{dy}{dx}$
- 2** **a** $2x + 2y \frac{dy}{dx} = 0$
 $2y \frac{dy}{dx} = -2x$
 $\frac{dy}{dx} = -\frac{x}{y}$
- b** $2 - \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$
 $2 = \frac{dy}{dx} (1 - 2y)$
 $\frac{dy}{dx} = \frac{2}{1 - 2y}$
- c** $4y^3 \frac{dy}{dx} = 2x - 6$
 $\frac{dy}{dx} = \frac{x - 3}{2y^3}$
- d** $2x + 2y \frac{dy}{dx} + 3 - 4 \frac{dy}{dx} = 0$
 $2x + 3 = \frac{dy}{dx} (4 - 2y)$
 $\frac{dy}{dx} = \frac{2x + 3}{4 - 2y}$
- e** $2x - 4y \frac{dy}{dx} + 1 + 3 \frac{dy}{dx} = 0$
 $2x + 1 = \frac{dy}{dx} (4y - 3)$
 $\frac{dy}{dx} = \frac{2x + 1}{4y - 3}$
- f** $\cos x - \frac{dy}{dx} \sin y = 0$
 $\cos x = \frac{dy}{dx} \sin y$
 $\frac{dy}{dx} = \frac{\cos x}{\sin y}$
- g** $6e^{3x} - 2e^{-2y} \frac{dy}{dx} = 0$
 $6e^{3x} = 2e^{-2y} \frac{dy}{dx}$
 $\frac{dy}{dx} = \frac{3e^{3x}}{e^{-2y}} = 3e^{3x + 2y}$
- h** $\sec^2 x - 2 \frac{dy}{dx} \operatorname{cosec} 2y \cot 2y = 0$
 $\sec^2 x = 2 \frac{dy}{dx} \operatorname{cosec} 2y \cot 2y$
 $\frac{dy}{dx} = \frac{\sec^2 x}{2 \operatorname{cosec} 2y \cot 2y}$
- i** $\frac{1}{x - 2} = \frac{2}{2y + 1} \frac{dy}{dx}$
 $\frac{dy}{dx} = \frac{2y + 1}{2(x - 2)}$
- 3** **a** $= 1 \times y + x \times \frac{dy}{dx}$
 $= y + x \frac{dy}{dx}$
- b** $= 2x \times y^3 + x^2 \times 3y^2 \frac{dy}{dx}$
 $= 2xy^3 + 3x^2y^2 \frac{dy}{dx}$
- c** $= \cos x \times \tan y + \sin x \times \frac{dy}{dx} \sec^2 y$
 $= \cos x \tan y + \frac{dy}{dx} \sin x \sec^2 y$
- d** $= 3(x - 2y)^2 \times (1 - 2 \frac{dy}{dx})$
 $= 3(x - 2y)^2 (1 - 2 \frac{dy}{dx})$

$$4 \quad \mathbf{a} \quad 2x \times y + x^2 \frac{dy}{dx} = 0$$

$$x^2 \frac{dy}{dx} = -2xy$$

$$\frac{dy}{dx} = -\frac{2y}{x}$$

$$\mathbf{c} \quad 8x - 2 \times y - 2x \times \frac{dy}{dx} + 6y \frac{dy}{dx} = 0$$

$$8x - 2y = \frac{dy}{dx} (2x - 6y)$$

$$\frac{dy}{dx} = \frac{4x - y}{x - 3y}$$

$$\mathbf{e} \quad \frac{dy}{dx} = 2(x + y) \times \left(1 + \frac{dy}{dx}\right)$$

$$\frac{dy}{dx} [1 - 2(x + y)] = 2(x + y)$$

$$\frac{dy}{dx} = \frac{2(x + y)}{1 - 2(x + y)}$$

$$\mathbf{g} \quad 2 \times y^2 + 2x \times 2y \frac{dy}{dx} - 3x^2 \times y - x^3 \times \frac{dy}{dx} = 0 \quad \mathbf{h} \quad 2y \frac{dy}{dx} + 1 \times \ln y + x \times \frac{1}{y} \frac{dy}{dx} = 0$$

$$2y^2 - 3x^2y = \frac{dy}{dx} (x^3 - 4xy)$$

$$\frac{dy}{dx} = \frac{2y^2 - 3x^2y}{x^3 - 4xy}$$

$$\frac{dy}{dx} \left(2y + \frac{x}{y}\right) = -\ln y$$

$$\frac{dy}{dx} = -\frac{\ln y}{2y + \frac{x}{y}} = -\frac{y \ln y}{2y^2 + x}$$

$$\mathbf{i} \quad 1 \times \sin y + x \times \frac{dy}{dx} \cos y + 2x \times \cos y + x^2 \times (-\sin y) \frac{dy}{dx} = 0$$

$$\sin y + 2x \cos y = \frac{dy}{dx} (x^2 \sin y - x \cos y)$$

$$\frac{dy}{dx} = \frac{\sin y + 2x \cos y}{x^2 \sin y - x \cos y}$$

$$5 \quad \mathbf{a} \quad 2x + 2y \frac{dy}{dx} - 3 \frac{dy}{dx} = 0$$

$$2x = \frac{dy}{dx} (3 - 2y)$$

$$\frac{dy}{dx} = \frac{2x}{3 - 2y}$$

$$\text{grad} = 4$$

$$\therefore y - 1 = 4(x - 2)$$

$$[y = 4x - 7]$$

$$\mathbf{b} \quad 4x - 1 \times y - x \times \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$4x - y = \frac{dy}{dx} (x - 2y)$$

$$\frac{dy}{dx} = \frac{4x - y}{x - 2y}$$

$$\text{grad} = -1$$

$$\therefore y - 5 = -(x - 3)$$

$$[y = 8 - x]$$

$$\mathbf{c} \quad 4 \frac{dy}{dx} \cos y - \sec x \tan x = 0$$

$$4 \frac{dy}{dx} \cos y = \sec x \tan x$$

$$\frac{dy}{dx} = \frac{\sec x \tan x}{4 \cos y}$$

$$\text{grad} = \frac{2 \times \frac{\sqrt{3}}{2}}{4 \times \frac{\sqrt{3}}{2}} = 1$$

$$\therefore y - \frac{\pi}{6} = x - \frac{\pi}{3}$$

$$[y = x - \frac{\pi}{6}]$$

$$\mathbf{d} \quad 2 \sec^2 x \times \cos y + 2 \tan x \times (-\sin y) \frac{dy}{dx} = 0$$

$$2 \sec^2 x \cos y = 2 \frac{dy}{dx} \tan x \sin y$$

$$\frac{dy}{dx} = \frac{\sec^2 x \cos y}{\tan x \sin y}$$

$$\text{grad} = \frac{2 \times \frac{1}{2}}{1 \times \frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2}{3} \sqrt{3}$$

$$\therefore y - \frac{\pi}{3} = \frac{2}{3} \sqrt{3} \left(x - \frac{\pi}{4}\right)$$

$$[4\sqrt{3}x - 6y + \pi(2 - \sqrt{3}) = 0]$$

$$6 \quad \mathbf{a} \quad 2x + 4y \frac{dy}{dx} - 1 + 4 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (4y + 4) = 1 - 2x$$

$$\frac{dy}{dx} = \frac{1-2x}{4(y+1)}$$

$$\mathbf{b} \quad \text{grad} = \frac{1}{8}$$

$$\therefore \text{grad of normal} = -8$$

$$\therefore y + 3 = -8(x - 1)$$

$$[y = 5 - 8x]$$

$$7 \quad \mathbf{a} \quad 2x + 4 \times y + 4x \times \frac{dy}{dx} - 6y \frac{dy}{dx} = 0$$

$$2x + 4y = \frac{dy}{dx} (6y - 4x)$$

$$\frac{dy}{dx} = \frac{x+2y}{3y-2x}$$

$$\text{grad} = -4$$

$$\therefore y - 2 = -4(x - 4)$$

$$[y = 18 - 4x]$$

$$\mathbf{b} \quad \text{at } Q, \frac{x+2y}{3y-2x} = -4$$

$$x + 2y = -4(3y - 2x)$$

$$x = 2y$$

sub. into equation of curve

$$\Rightarrow (2y)^2 + 4y(2y) - 3y^2 = 36$$

$$y^2 = 4$$

$$y = 2 \text{ (at } P) \text{ or } -2$$

$$\therefore Q(-4, -2)$$

$$8 \quad \ln y = \ln a^x$$

$$\ln y = x \ln a$$

$$\frac{1}{y} \frac{dy}{dx} = \ln a$$

$$\frac{dy}{dx} = y \ln a = a^x \ln a$$

$$9 \quad \mathbf{a} = 3^x \ln 3$$

$$\mathbf{b} = 6^{2x} \ln 6 \times 2 \\ = 2(6^{2x}) \ln 6$$

$$\mathbf{c} = 5^{1-x} \ln 5 \times (-1) \\ = -(5^{1-x}) \ln 5$$

$$\mathbf{d} = 2^{x^3} \ln 2 \times 3x^2 \\ = 3x^2(2^{x^3}) \ln 2$$

$$10 \quad \frac{dN}{dt} = 800(1.04)^t \times \ln 1.04$$

$$N = 4000$$

$$\therefore 4000 = 800(1.04)^t$$

$$(1.04)^t = 5$$

$$\frac{dN}{dt} = 800 \times 5 \times \ln 1.04 = 157 \text{ (3sf)}$$

\therefore growing at rate of 157 per minute