C4 DIFFERENTIATION Worksheet B

1 A curve is given by the parametric equations

$$x = 2 + t, \quad y = t^2 - 1.$$

a Write down expressions for $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

b Hence, show that $\frac{dy}{dx} = 2t$.

2 Find and simplify an expression for $\frac{dy}{dx}$ in terms of the parameter t in each case.

a $x = t^2, \quad y = 3t$

b $x = t^2 - 1, \quad y = 2t^3 + t^2$

c $x = 2 \sin t, \quad y = 6 \cos t$

d $x = 3t - 1, \quad y = 2 - \frac{1}{t}$

e $x = \cos 2t, \quad y = \sin t$

f $x = e^{t+1}, \quad y = e^{2t-1}$

g $x = \sin^2 t, \quad y = \cos^3 t$

h $x = 3 \sec t, \quad y = 5 \tan t$

i $x = \frac{1}{t+1}, \quad y = \frac{t}{t-1}$

3 Find, in the form $y = mx + c$, an equation for the tangent to the given curve at the point with the given value of the parameter t.

a $x = t^3, \quad y = 3t^2, \qquad t = 1$

b $x = 1 - t^2, \quad y = 2t - t^2, \qquad t = 2$

c $x = 2 \sin t, \quad y = 1 - 4 \cos t, \qquad t = \frac{\pi}{3}$

d $x = \ln(4 - t), \quad y = t^2 - 5, \qquad t = 3$

4 Show that the normal to the curve with parametric equations

$$x = \sec \theta, \quad y = 2 \tan \theta, \quad 0 \le \theta < \tfrac{\pi}{2},$$

at the point where $\theta = \frac{\pi}{3}$, has the equation

$$\sqrt{3}\,x + 4y = 10\sqrt{3}.$$

5 A curve is given by the parametric equations

$$x = \frac{1}{t}, \quad y = \frac{1}{t+2}.$$

a Show that $\frac{dy}{dx} = \left(\frac{t}{t+2}\right)^2$.

b Find an equation for the normal to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

6 A curve has parametric equations

$$x = \sin 2t, \quad y = \sin^2 t, \quad 0 \le t < \pi.$$

a Show that $\frac{dy}{dx} = \frac{1}{2} \tan 2t$.

b Find an equation for the tangent to the curve at the point where $t = \frac{\pi}{6}$.

7 A curve has parametric equations

$$x = 3 \cos \theta, \quad y = 4 \sin \theta, \quad 0 \le \theta < 2\pi.$$

a Show that the tangent to the curve at the point $(3 \cos \alpha, 4 \sin \alpha)$ has the equation

$$3y \sin \alpha + 4x \cos \alpha = 12.$$

b Hence find an equation for the tangent to the curve at the point $(-\frac{3}{2}, 2\sqrt{3})$.

8 A curve is given by the parametric equations

$$x = t^2, \quad y = t(t - 2), \quad t \geq 0.$$

a Find the coordinates of any points where the curve meets the coordinate axes.

b Find $\frac{dy}{dx}$ in terms of x

i by first finding $\frac{dy}{dx}$ in terms of t,

ii by first finding a cartesian equation for the curve.

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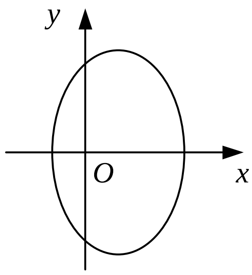

The diagram shows the ellipse with parametric equations

$$x = 1 - 2\cos\theta, \quad y = 3\sin\theta, \quad 0 \leq \theta < 2\pi.$$

a Find $\frac{dy}{dx}$ in terms of θ.

b Find the coordinates of the points where the tangent to the curve is

i parallel to the x-axis,

ii parallel to the y-axis.

10 A curve is given by the parametric equations

$$x = \sin\theta, \quad y = \sin 2\theta, \quad 0 \leq \theta \leq \tfrac{\pi}{2}.$$

a Find the coordinates of any points where the curve meets the coordinate axes.

b Find an equation for the tangent to the curve that is parallel to the x-axis.

c Find a cartesian equation for the curve in the form $y = f(x)$.

11 A curve has parametric equations

$$x = \sin^2 t, \quad y = \tan t, \quad -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}.$$

a Show that the tangent to the curve at the point where $t = \frac{\pi}{4}$ passes through the origin.

b Find a cartesian equation for the curve in the form $y^2 = f(x)$.

12 A curve is given by the parametric equations

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}, \quad t \neq 0.$$

a Find an equation for the tangent to the curve at the point P where $t = 3$.

b Show that the tangent to the curve at P does not meet the curve again.

c Show that the cartesian equation of the curve can be written in the form

$$x^2 - y^2 = k,$$

where k is a constant to be found.