

C4

DIFFERENTIATION

Worksheet A

- 1 A curve is given by the parametric equations

$$x = t^2 + 1, \quad y = \frac{4}{t}.$$

- a Write down the coordinates of the point on the curve where $t = 2$.
 b Find the value of t at the point on the curve with coordinates $(\frac{5}{4}, -8)$.

- 2 A curve is given by the parametric equations

$$x = 1 + \sin t, \quad y = 2 \cos t, \quad 0 \leq t < 2\pi.$$

- a Write down the coordinates of the point on the curve where $t = \frac{\pi}{2}$.
 b Find the value of t at the point on the curve with coordinates $(\frac{3}{2}, -\sqrt{3})$.

- 3 Find a cartesian equation for each curve, given its parametric equations.

a $x = 3t, \quad y = t^2$

b $x = 2t, \quad y = \frac{1}{t}$

c $x = t^3, \quad y = 2t^2$

d $x = 1 - t^2, \quad y = 4 - t$

e $x = 2t - 1, \quad y = \frac{2}{t^2}$

f $x = \frac{1}{t-1}, \quad y = \frac{1}{2-t}$

- 4 A curve has parametric equations

$$x = 2t + 1, \quad y = t^2.$$

- a Find a cartesian equation for the curve.
 b Hence, sketch the curve.

- 5 Find a cartesian equation for each curve, given its parametric equations.

a $x = \cos \theta, \quad y = \sin \theta$

b $x = \sin \theta, \quad y = \cos 2\theta$

c $x = 3 + 2 \cos \theta, \quad y = 1 + 2 \sin \theta$

d $x = 2 \sec \theta, \quad y = 4 \tan \theta$

e $x = \sin \theta, \quad y = \sin^2 2\theta$

f $x = \cos \theta, \quad y = \tan^2 \theta$

- 6 A circle has parametric equations

$$x = 1 + 3 \cos \theta, \quad y = 4 + 3 \sin \theta, \quad 0 \leq \theta < 2\pi.$$

- a Find a cartesian equation for the circle.
 b Write down the coordinates of the centre and the radius of the circle.
 c Sketch the circle and label the points on the circle where θ takes each of the following values:

$$0, \quad \frac{\pi}{4}, \quad \frac{\pi}{2}, \quad \frac{3\pi}{4}, \quad \pi, \quad \frac{5\pi}{4}, \quad \frac{3\pi}{2}, \quad \frac{7\pi}{4}.$$

- 7 Write down parametric equations for a circle

a centre $(0, 0)$, radius 5,

b centre $(6, -1)$, radius 2,

c centre (a, b) , radius r , where a, b and r are constants and $r > 0$.

- 8 For each curve given by parametric equations, find a cartesian equation and hence, sketch the curve, showing the coordinates of any points where it meets the coordinate axes.

a $x = 2t, \quad y = 4t(t-1)$

b $x = 1 - \sin \theta, \quad y = 2 - \cos \theta, \quad 0 \leq \theta < 2\pi$

c $x = t - 3, \quad y = 4 - t^2$

d $x = t + 1, \quad y = \frac{2}{t}$