C4 Essentials: *Summary of AQA Core 4 content not provided in the formula book*

Binomial expansion

Unless n is a positive integer, the expansion of $(1+kx)^n$ will have an infinite number of terms.

It will be valid only for $|kx|<1$ (that is, $|x|<\frac{1}{k}$).

$$(a+bx)^n = a^n\left(1+\frac{b}{a}x\right)^n$$

Factor theorem

$(ax-b)$ is a factor $\iff$ $\frac{b}{a}$ is a root

Remainder theorem

$P(x)\div(ax-b)$ has remainder $R \iff P\left(\frac{b}{a}\right)=R$

Integration for partial fractions

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\int \frac{a}{ax+b}\,dx = \frac{1}{a}\ln|ax+b|+C$$

$$\int \frac{1}{(ax+b)^2}\,dx = -\frac{1}{a(ax+b)}+C$$

Implicit differentiation

$\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}$	$\frac{d}{dx}(\ln y) = \frac{1}{y}\frac{dy}{dx}$
$\frac{d}{dx}(\sin y) = \cos y\frac{dy}{dx}$	$\frac{d}{dx}[f(y)] = f'(y)\frac{dy}{dx}$

Parametric equations

$$\frac{dy}{dx} = \frac{dy}{dt}\div\frac{dx}{dt}$$

Double angle formulae

$$\sin 2A = 2\sin A\cos A \qquad \tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$or \quad \cos 2A = 1-2\sin^2 A \quad or \quad \cos 2A = 2\cos^2 A - 1$$

$R\cos(x\pm\alpha)$ and $R\sin(x\pm\alpha)$

How to rewrite expressions of the form $a\sin x \pm b\cos x$:

	Method:	**Example: $R\cos(x-\alpha)$**
1.	Calculate R using $R=\sqrt{a^2+b^2}$.	$R=\sqrt{a^2+b^2}$
2.	Rewrite $\cos(x\pm\alpha)$ or $\sin(x\pm\alpha)$ using $\cos(A\pm B)$ or $\sin(A\pm B)$.	$\cos(x-\alpha)$ $=\cos x\cos\alpha+\sin x\sin\alpha$
3.	Compare coefficients of $\sin x$ and $\cos x$.	$a=R\sin\alpha$ & $b=R\cos\alpha$
4.	Rearrange to find α.	$\alpha=\sin^{-1}\frac{a}{R}$ & $\alpha=\cos^{-1}\frac{b}{R}$

Exponential growth & decay

$$\frac{dP}{dt}=kP \Rightarrow \int\frac{1}{P}\,dP=\int k\,dt \Rightarrow \ln P = kt+C \Rightarrow P=e^{kt+C}=e^Ce^{kt}=Ae^{kt}$$

$$P=Ae^{kt} \Rightarrow \frac{dP}{dt}=kAe^{kt}=kP$$

Vectors

$$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$$

$$\boldsymbol{a}\cdot\boldsymbol{b}=|\boldsymbol{a}||\boldsymbol{b}|\cos\theta$$

$$\left\|\begin{bmatrix}a\\b\\c\end{bmatrix}\right\|=\sqrt{a^2+b^2+c^2}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}\cdot\begin{bmatrix}p\\q\\r\end{bmatrix}=xp+yq+zr$$

Vector line: $\boldsymbol{r}=\begin{bmatrix}x\\y\\z\end{bmatrix}+\lambda\begin{bmatrix}p\\q\\r\end{bmatrix}$ where $\begin{bmatrix}x\\y\\z\end{bmatrix}$ is the position and $\begin{bmatrix}p\\q\\r\end{bmatrix}$ is the direction.

$\boldsymbol{a}$ *and* $\boldsymbol{b}$ *parallel* $\Rightarrow \boldsymbol{a}=k\boldsymbol{b}$ | *Skew* $\Rightarrow$ *Don't cross, aren't parallel*