C4 Integration

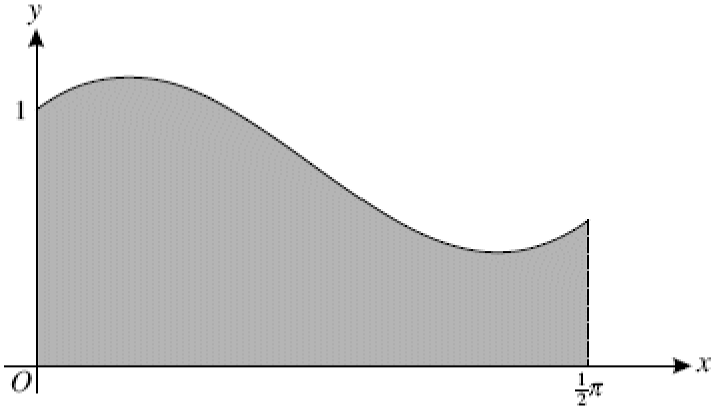
1. June 2010 qu. 4

Use the substitution $u = \sqrt{x+2}$ to find the exact value of $\int_{-1}^{7} \frac{x^2}{\sqrt{x+2}}\,dx$. [7]

2. June 2010 qu. 9

(i) Find $\int (x + \cos 2x)^2\,dx$. [9]

(ii)



The diagram shows the part of the curve $y = x + \cos 2x$ for $0 \le x \le \frac{1}{2}\pi$. The shaded region bounded by the curve, the axes and the line $x = \frac{1}{2}\pi$ is rotated completely about the x-axis to form a solid of revolution of volume V. Find V, giving your answer in an exact form.

3. Jan 2010 qu. 3

By expressing $\cos 2x$ in terms of $\cos x$, find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \frac{\cos 2x}{\cos^2 x}\,dx$. [5]

4. Jan 2010 qu. 4

Use the substitution $u = 2 + \ln t$ to find the exact value of $\int_{1}^{e} \frac{1}{t(2+\ln t)^2}\,dt$. [6]

5. Jan 2010 qu. 8

(i) State the derivative of $e^{\cos x}$. [1]

(ii) Hence use integration by parts to find the exact value of $\int_{0}^{\frac{1}{2}\pi} \cos x \sin x\, e^{\cos x}\,dx$. [6]

6. June 2009 qu. 2

Use the substitution $x = \tan\theta$ to find the exact value of $\int_{1}^{\sqrt{3}} \frac{1-x^2}{1+x^2}\,dx$. [7]

7. June 2009 qu. 4

(i) Differentiate $e^x(\sin 2x - 2\cos 2x)$, simplifying your answer. [4]

(ii) Hence find the exact value of $\int_{0}^{\frac{1}{4}\pi} e^x \sin 2x\,dx$. [3]

8. June 2009 qu. 6

The expression $\dfrac{4x}{(x-5)(x-3)^2}$ is denoted by f(x).

(i) Express f(x) in the form $\dfrac{A}{x-5}+\dfrac{B}{x-3}+\dfrac{C}{(x-3)^2}$, where A, B and C are constants. [4]

(ii) Hence find the exact value of $\int_1^2 \mathrm{f}(x)\,\mathrm{d}x$. [5]

9. Jan 2009 qu. 2

Find $\int x\sec^2 x\,\mathrm{d}x$. [4]

10. Jan 2009 qu. 4

Find the exact value of $\int_0^{\frac{1}{4}\pi}(1+\sin x)^2\;\mathrm{d}x$. [6]

11. Jan 2009 qu. 5

(i) Show that the substitution $u=\sqrt{x}$ transforms $\int \dfrac{1}{x(1+\sqrt{x})}\mathrm{d}x$ to $\int \dfrac{2}{u(1+u)}\mathrm{d}u$. [3]

(ii) Hence find the exact value of $\int_1^9 \dfrac{1}{x(1+\sqrt{x})}\mathrm{d}x$. [5]

12. June 2008 qu. 2

Find the exact value of $\int_1^{\mathrm{e}} x^4\;\ln x\,\mathrm{d}x$. [5]

13. June 2008 qu. 8

(i) Given that $\dfrac{2t}{(t+1)^2}$ can be expressed in the form $\dfrac{A}{t+1}+\dfrac{B}{(t+1)^2}$, find the values of the constants A and B. [3]

(ii) Show that the substitution $t=\sqrt{2x-1}$ transforms $\int \dfrac{1}{x+\sqrt{2x-1}}\mathrm{d}x$ to $\int \dfrac{2t}{(t+1)^2}\,\mathrm{d}t$. [4]

(iii) Hence find the exact value of $\int_1^5 \dfrac{1}{x+\sqrt{2x-1}}\mathrm{d}x$. [4]

14. Jan 2008 qu. 2

(i) Express $\dfrac{x}{(x+1)(x+2)}$ in partial fractions. [3]

(ii) Hence find $\int \dfrac{x}{(x+1)(x+2)}\mathrm{d}x$. [2]

15. Jan 2008 qu. 7

(i) Given that $A(\sin\theta+\cos\theta)+B(\cos\theta-\sin\theta)\equiv 4\sin\theta$, find the values of the constants A and B. [3]

(ii) Hence find the exact value of $\int_0^{\frac{1}{4}\pi}\dfrac{4\sin\theta}{\sin\theta+\cos\theta}\mathrm{d}\theta$. giving your answer in the form $a\pi-\ln b$. [5]

16. Jan 2008 qu.10

(i) Use the substitution $x = \sin\theta$ to find the exact value of $\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx$. [6]

(ii) Find the exact value of $\int_1^3 \frac{\ln x}{x^2}\,dx$. [5]

17. June 2007 qu. 2

Find the exact value of $\int_0^1 x^2 e^x\,dx$. [6]

18. June 2007 qu. 3

Find the exact volume generated when the region enclosed between the x-axis and the portion of the curve $y = \sin x$ between $x = 0$ and $x = \pi$ is rotated completely about the x-axis. [6]

19. Jan 2007 qu. 7

(i) Find the quotient and the remainder when $2x^3 + 3x^2 + 9x + 12$ is divided by $x^2 + 4$. [4]

(ii) Hence express in the form $\frac{2x^3+3x^2+9x+12}{x^2+4}$ in the form $Ax + B + \frac{Cx+D}{x^2+4}$, where the values of the constants A, B, C and D are to be stated. [1]

(iii) Use the result of part (ii) to find the exact value of $\int_1^3 \frac{2x^3+3x^2+9x+12}{x^2+4}\,dx$. [5]

20. Jan 2007 qu. 2

Find the exact value of $\int_1^2 x\ln x\,dx$ [5]

21. Jan 2007 qu. 4

Use the substitution $u = 2x - 5$ to show that $\int_{\frac{5}{2}}^3 (4x-8)(2x-5)^7\,dx = \frac{17}{72}$. [5]

22. June 2006 qu. 3

(i) Express $\frac{3-2x}{x(3-x)}$ in partial fractions. [3]

(ii) Show that $\int_1^2 \frac{3-2x}{x(3-x)}\,dx = 0$. [4]

(iii) What does the result of part (ii) indicate about the graph of $y = \frac{3-2x}{x(3-x)}$ between $x = 1$ and $x = 2$? [1]

23. June 2006 qu. 6

(i) Show that the substitution $u = e^x + 1$ transforms $\int \frac{e^{2x}}{e^x+1}\,dx$ to $\int \frac{u-1}{u}\,du$. [3]

(ii) Hence show that $\int_0^1 \frac{e^{2x}}{e^x+1}\,dx = e - 1 - \ln\left(\frac{e+1}{2}\right)$. [5]

24. June 2006 qu. 8

(i) Show that $\int \cos^2 6x\,dx = \frac{1}{2}x + \frac{1}{24}\sin 12x + c$. [3]

(ii) Hence find the exact value of $\int_0^{\frac{1}{12}\pi} x\cos^2 6x\,dx$. [6]

25. Jan 2006 qu. 4

(i) Use integration by parts to find $\int x \sec^2 x \, dx$. [4]

(ii) Hence find $\int x \tan^2 x \, dx$. [3]

26. Jan 2006 qu. 6

(i) Show that the substitution $x = \sin^2 \theta$ transforms $\int \sqrt{\frac{x}{1-x}} \, dx$ to $\int 2\sin^2\theta \, d\theta$. [4]

(ii) Hence find $\int_0^1 \sqrt{\frac{x}{1-x}} dx$. [5]

27. June 2005 qu. 2

Evaluate $\int_0^{\frac{1}{2}\pi} x \cos x \, dx$, giving your answer in an exact form.

[5]

28. June 2005 qu. 4

(i) Show that the substitution $x = \tan \theta$ transforms $\int \frac{1}{(1+x^2)^2} dx$ to $\int \cos^2\theta \, d\theta$. [3]

(ii) Hence find the exact value of $\int_0^1 \frac{1}{(1+x^2)^2} dx$. [4]