1(i) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -8-3\lambda \\ -2 \\ 6+\lambda \end{pmatrix}$ Substituting into plane equation: $2(-8-3\lambda) - 3(-2) + 6 + \lambda = 11$ $\Rightarrow$ $-16 - 6\lambda + 6 + 6 + \lambda = 11$ $\Rightarrow$ $5\lambda = -15,\ \lambda = -3$ So point of intersection is (1, –2, 3)	B1 M1 A1 A1ft [4]	
(ii) Angle between $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$ $\cos\theta = \frac{2\times(-3)+(-3)\times 0+1\times 1}{\sqrt{14}\sqrt{10}}$ = (–)0.423 $\Rightarrow$ acute angle = 65°	B1 M1 A1 A1 [4]	 allow M1 for a complete method only for any vectors

2	**(i)**	$\overrightarrow{AB} = \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} -1 \\ -11 \\ 3 \end{pmatrix}$	B1 B1 **[2]**	
	(ii)	$\mathbf{n}.\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}.\begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix} = -4+1+3=0$ $\mathbf{n}.\overrightarrow{AC} = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}.\begin{pmatrix} -1 \\ -11 \\ 3 \end{pmatrix} = -2+11-9=0$ $\Rightarrow$ plane is $2x - y - 3z = d$ $x = 1, y = 3, z = -2 \Rightarrow d = 2 - 3 + 6 = 5$ $\Rightarrow$ plane is $2x - y - 3z = 5$	M1 E1 E1 M1 A1 **[5]**	scalar product

3(i) Normal vectors $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ Angle between planes is θ, where $\cos\theta = \dfrac{2\times1+(-1)\times0+1\times(-1)}{\sqrt{2^2+(-1)^2+1^2}\sqrt{1^2+0^2+(-1)^2}}$ $= 1/\sqrt{12}$ $\Rightarrow$ $\theta = 73.2°$ or 1.28 rads	B1 M1 M1 A1 [4]	 scalar product finding invcos of scalar product divided by two modulae
(ii) $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 2+2\lambda \\ -\lambda \\ 1+\lambda \end{pmatrix}$ $\Rightarrow$ $2(2 + 2\lambda) - (-\lambda) + (1 + \lambda) = 2$ $\Rightarrow$ $5 + 6\lambda = 2$ $\Rightarrow$ $\lambda = -½$ So point of intersection is (1, ½ , ½)	B1 M1 A1 A1 [4]	

4 (i) Plane has equation $x - y + 2z = c$	B1	$x - y + 2z = c$
At $(2, -1, 4)$, $2 + 1 + 8 = c$	M1	finding c
$\Rightarrow c = 11$.	A1	
(ii) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7+\lambda \\ 12+3\lambda \\ 9+2\lambda \end{pmatrix}$	M1	
$\Rightarrow \quad 7 + \lambda - (12 + 3\lambda) + 2(9 + 2\lambda) = 11$	M1	ft their equation from (i)
$\Rightarrow \quad 2\lambda = -2$		
$\Rightarrow \quad \lambda = -1$	A1	ft their x-y+2z=c
Coordinates are (6, 9, 7)	A1	cao
	[7]	

5 (i) AE = $\sqrt{(15^2 + 20^2 + 0^2)} = 25$	M1 A1 [2]	
(ii) $\overrightarrow{AE} = \begin{pmatrix}15\\-20\\0\end{pmatrix} = 5\begin{pmatrix}3\\-4\\0\end{pmatrix}$ Equation of BD is $\mathbf{r} = \begin{pmatrix}-1\\-7\\11\end{pmatrix} + \lambda\begin{pmatrix}3\\-4\\0\end{pmatrix}$ BD = 15 $\Rightarrow \lambda = 3$ $\Rightarrow$ D is (8, –19, 11)	M1 A1 M1 A1cao [4]	Any correct form or $\mathbf{r} = \begin{pmatrix}-1\\-7\\11\end{pmatrix} + \lambda\begin{pmatrix}15\\-20\\0\end{pmatrix}$ λ = 3 or 3/5 as appropriate
(iii) At A: –3 × 0 + 4 × 0 + 5 × 6 = 30 At B: –3 × (–1) + 4 × (–7) + 5 × 11 = 30 At C: –3 × (–8) + 4 × (–6) + 5 × 6 = 30 Normal is $\begin{pmatrix}-3\\4\\5\end{pmatrix}$	M1 A2,1,0 B1 [4]	One verification (OR B1 Normal, M1 scalar product with 1 vector in the plane, A1two correct, A1 verification with a point OR M1 vector form of equation of plane eg r=0i+0j+6k+μ(i+7j-5k)+ν(8i+6j+0k) M1 elimination of both parameters A1 equation of plane B1 Normal *)
(iv) $\begin{pmatrix}4\\3\\5\end{pmatrix}.\overrightarrow{AE} = \begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}15\\-20\\0\end{pmatrix} = 60 - 60 = 0$ $\begin{pmatrix}4\\3\\5\end{pmatrix}.\overrightarrow{AB} = \begin{pmatrix}4\\3\\5\end{pmatrix}\begin{pmatrix}-1\\-7\\5\end{pmatrix} = -4 - 21 + 25 = 0$ $\Rightarrow \begin{pmatrix}4\\3\\5\end{pmatrix}$ is normal to plane Equation is 4x + 3y + 5z = 30.	M1 E1 M1 A1 [4]	scalar product with one vector in plane = 0 scalar product with another vector in plane = 0 4x + 3y + 5z = … 30 OR as * above OR M1 for subst 1 point in 4x+3y+5z= ,A1 for subst 2 further points =30 A1 correct equation , B1 Normal
(v) Angle between planes is angle between normals $\begin{pmatrix}4\\3\\5\end{pmatrix}$ and $\begin{pmatrix}-3\\4\\5\end{pmatrix}$ $\cos\theta = \dfrac{4\times(-3)+3\times4+5\times5}{\sqrt{50}\times\sqrt{50}} = \dfrac{1}{2}$ $\Rightarrow \theta = 60°$	M1 M1 A1 A1 [4]	 Correct method for any 2 vectors their normals only (rearranged) or 120° cao