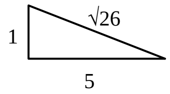
1(i) At A: $3\times0 + 2\times0 + 20\times(-15) + 300 = 0$ At B: $3\times100 + 2\times0 + 20\times(-30) + 300 = 0$ At C: $3\times0 + 2\times100 + 20\times(-25) + 300 = 0$ So ABC has equation $3x + 2y + 20z + 300 = 0$	M1 A2,1,0 [3]	substituting co-ords into equation of plane… for ABC **OR** using two vectors in the plane form vector product M1A1 then $3x + 2y + 20z = c = -300$ A1 **OR** using vector equation of plane M1,elim both parameters M1, A1
(ii) $\overrightarrow{DE} = \begin{pmatrix}100\\0\\-10\end{pmatrix}$ $\overrightarrow{DF} = \begin{pmatrix}0\\100\\5\end{pmatrix}$ $\begin{pmatrix}100\\0\\-10\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\20\end{pmatrix} = 100\times2 + 0\times-1 + -10\times20 = 200 - 200 = 0$ $\begin{pmatrix}0\\100\\5\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\20\end{pmatrix} = 0\times2 + 100\times-1 + 5\times20 = -100 + 100 = 0$ Equation of plane is $2x - y + 20z = c$ At D (say) $c = 20\times-40 = -800$ So equation is $2x - y + 20z + 800 = 0$	B1B1 B1 B1 M1 A1 [6]	 need evaluation need evaluation
(iii) Angle is θ, where $\cos\theta = \dfrac{\begin{pmatrix}2\\-1\\20\end{pmatrix}\cdot\begin{pmatrix}3\\2\\20\end{pmatrix}}{\sqrt{2^2+(-1)^2+20^2}\sqrt{3^2+2^2+20^2}} = \dfrac{404}{\sqrt{405}\sqrt{413}}$ $\Rightarrow$ $\theta = 8.95°$	M1 A1 A1 A1cao [4]	formula with correct vectors top bottom (or 0.156 radians)
(iv) RS: $\mathbf{r} = \begin{pmatrix}15\\34\\0\end{pmatrix} + \lambda\begin{pmatrix}3\\2\\20\end{pmatrix} = \begin{pmatrix}15+3\lambda\\34+2\lambda\\20\lambda\end{pmatrix}$ $\Rightarrow$ $3(15+3\lambda) + 2(34+2\lambda) + 20.20\lambda + 300 = 0$ $\Rightarrow$ $45 + 9\lambda + 68 + 4\lambda + 400\lambda + 300 = 0$ $\Rightarrow$ $413 + 413\lambda = 0$ $\Rightarrow$ $\lambda = -1$ so S is (12, 32, –20)	B1 B1 M1 A1 A1 [5]	$\begin{pmatrix}15\\34\\0\end{pmatrix} + \ldots$ $\ldots + \lambda\begin{pmatrix}3\\2\\20\end{pmatrix}$ solving with plane $\lambda = -1$ cao

2 Normal vectors are $\begin{pmatrix}2\\3\\4\end{pmatrix}$ and $\begin{pmatrix}1\\-2\\1\end{pmatrix}$ $\Rightarrow \begin{pmatrix}2\\3\\4\end{pmatrix}.\begin{pmatrix}1\\-2\\1\end{pmatrix} = 2-6+4=0$ $\Rightarrow$ planes are perpendicular.	B1 B1 M1 E1 [4]	

3 $\mathbf{r}=\begin{pmatrix}1\\2\\-1\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\3\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1-\lambda\\2+2\lambda\\-1+3\lambda\end{pmatrix}$		
When $x = -1$, $1 - \lambda = -1$, $\Rightarrow \lambda = 2$	M1	Finding λ or μ
$\Rightarrow y = 2 + 2\lambda = 6$, $z = -1 + 3\lambda = 5$ $\Rightarrow$ point lies on first line	E1	checking other two coordinates
$\mathbf{r}=\begin{pmatrix}0\\6\\3\end{pmatrix}+\mu\begin{pmatrix}1\\0\\-2\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}\mu\\6\\3-2\mu\end{pmatrix}$ When $x = -1$, $\mu = -1$, $\Rightarrow y = 6$, $z = 3 - 2\mu = 5$ $\Rightarrow$ point lies on second line	E1	checking other two co-ordinates
Angle between $\begin{pmatrix}-1\\2\\3\end{pmatrix}$ and $\begin{pmatrix}1\\0\\-2\end{pmatrix}$ is θ, where	M1	Finding angle between correct vectors
$\cos\theta = \dfrac{-1\times1+2\times0+3\times-2}{\sqrt{14}.\sqrt{5}}$	M1	use of formula
$= -\dfrac{7}{\sqrt{70}}$	A1	$\pm\dfrac{7}{\sqrt{70}}$
$\Rightarrow \quad \theta = 146.8°$ $\Rightarrow$ acute angle is 33.2°	A1cao [7]	Final answer must be acute angle

4 (i) P is (0, 10, 30) Q is (0, 20, 15) R is (–15, 20, 30) $\Rightarrow \overrightarrow{PQ} = \begin{pmatrix} 0-0 \\ 20-10 \\ 15-30 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ -15 \end{pmatrix}$ * $\Rightarrow \overrightarrow{PR} = \begin{pmatrix} -15-0 \\ 20-10 \\ 30-30 \end{pmatrix} = \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix}$ *	**B2,1,0** **E1** **E1** **[4]**	
(ii) $\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} . \begin{pmatrix} 0 \\ 10 \\ -15 \end{pmatrix} = 0 + 30 - 30 = 0$	**M1**	Scalar product with 1 vector in the plane OR vector x product oe
$\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} . \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix} = -30 + 30 + 0 = 0$	**E1**	
$\Rightarrow \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$ is normal to the plane		
$\Rightarrow$ equation of plane is $2x + 3y + 2z = c$	**M1**	$2x + 3y + 2z = c$ **or** an appropriate vector form
At P (say), $x = 0$, $y = 10$, $z = 30$ $\Rightarrow c = 2\times0 + 3\times10 + 2\times30 = 90$	**M1dep**	substituting to find c **or** completely eliminating parameters
$\Rightarrow$ equation of plane is $2x + 3y + 2z = 90$	**A1 cao** **[5]**	
(iii) S is $(-7\frac{1}{2}, 20, 22\frac{1}{2})$	**B1**	
$\overrightarrow{OT} = \overrightarrow{OP} + \frac{2}{3}\overrightarrow{PS}$	**M1**	**Or** $\frac{1}{3}(\overrightarrow{OP} + \overrightarrow{OR} + \overrightarrow{OQ})$ oe ft their S
$= \begin{pmatrix} 0 \\ 10 \\ 30 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} -7\frac{1}{2} \\ 10 \\ -7\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -5 \\ 16\frac{2}{3} \\ 25 \end{pmatrix}$	**A1ft**	**Or** $\frac{1}{3}\begin{pmatrix} 0 \\ 10 \\ 30 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} -7\frac{1}{2} \\ 20 \\ 22\frac{1}{2} \end{pmatrix}$ ft their S
So T is $(-5, 16\frac{2}{3}, 25)$ *	**E1** **[4]**	
(iv) $\mathbf{r} = \begin{pmatrix} -5 \\ 16\frac{2}{3} \\ 25 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$	**B1,B1**	$\begin{pmatrix} -5 \\ 16\frac{2}{3} \\ 25 \end{pmatrix} + \ldots \ldots + \lambda\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$
At C (–30, 0, 0): $-5 + 2\lambda = -30$, $16\frac{2}{3} + 3\lambda = 0$, $25 + 2\lambda = 0$	**M1** **A1**	Substituting coordinates of C into vector equation At least 2 relevant correct equations for λ
1st and 3rd eqns give $\lambda = -12\frac{1}{2}$, not compatible with 2nd. So line does not pass through C.	**E1** **[5]**	oe www

5	**(i)**		DE = $\sqrt{[(-5)^2 + 0^2 + 1^2]} = \sqrt{26}$	M1 A1	oe	
				M1	oe using scalar products eg –5**i** + **k** with **i** oe	
			$\cos\theta = 5/\sqrt{26}$ oe $\Rightarrow \quad \theta = 11.3°$	A1 **[4]**	or better (or 168.7°). Allow radians.	

Question			Answer	Marks	Guidance
5	(ii)		$\overrightarrow{AE} = \begin{pmatrix}1\\4\\3\end{pmatrix}$, $\overrightarrow{ED} = \begin{pmatrix}5\\0\\-1\end{pmatrix}$	B1	**two** relevant direction vectors (or 6**i** + 4**j** + 2**k** oe)
			$\begin{pmatrix}1\\4\\3\end{pmatrix}.\begin{pmatrix}1\\-4\\5\end{pmatrix} = 1-16+15=0$	B1	scalar product with a **direction** vector in the plane (including evaluation and = 0) (**OR** M1 forms vector cross product with at least two correct terms in solution)
			$\begin{pmatrix}5\\0\\-1\end{pmatrix}.\begin{pmatrix}1\\-4\\5\end{pmatrix} = 5+0-5=0$	B1	scalar product with second **direction** vector, with evaluation. (following **OR** above, A1 all correct ie a multiple of **i – 4j** + 5**k**) (NB finding only one direction vector and its scalar product is B1 only.)
			⇒ **i** – 4**j** + 5**k** is normal to AED ⇒ eqn of AED is $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}1\\-4\\5\end{pmatrix} = \begin{pmatrix}0\\-4\\0\end{pmatrix}.\begin{pmatrix}1\\-4\\5\end{pmatrix}$	M1	for $x - 4y + 5z = c$ oe
			⇒ $x - 4y + 5z = 16$	A1	M1A0 for **i** – 4**j** + 5**k** = 16
			B lies in plane if $8 - 4(-a) + 5.0 = 16$	M1	allow M1 for subst in their plane or if found from say scalar product of normal
			⇒ $a = 2$	A1	with vector EB can also get M1A1 **For first five marks above** **SC1**, if states, 'if **i** – 4**j** + 5**k** is normal then of form $x - 4y + 5z$ = c' and substitutes one coordinate gets M1A1, then substitutes other two coordinates A2 (not A1,A1).Then states so $\begin{pmatrix}1\\-4\\5\end{pmatrix}$ is normal can get B1 provided that there is a clear argument ie M1A1A2B1. Without a clear argument this is B0. **SC2,** if finds two relevant vectors, B1 and then finds equation of the plane from vector form, $r = a + \mu b + \lambda c$ gets B1. Eliminating parameters B1cao. If then states 'so $\begin{pmatrix}1\\-4\\5\end{pmatrix}$ is normal' can get B1 (4/5).
				[7]	

Question			Answer	Marks	Guidance
5	**(iii)**		D: 6 + 2 = 8 B: 8 + 0 = 8 C: 8 + 0 = 8 $\Rightarrow$ plane BCD is $x + z = 8$ Angle between **i** – 4**j** + 5**k** and **i** + **k** is θ $\Rightarrow$ cos θ = ×1+(–4)×0 × 1)/√42√2 /√84 $\Rightarrow$ θ = 49.1°	B2,1,0 M1 M1 A1 A1 **[6]**	or any valid method for finding $x + z = 8$ gets M1A1 between two correct relevant vectors complete method (including cosine) (**for M1** ft their normal(s) to their plane(s)) allow correct substitution or ±6/√84 , correct only or 0.857 radians (or better) acute only