1 With respect to cartesian coordinates Oxyz, a laser beam ABC is fired from the point A(1, 2, 4), and is reflected at point B off the plane with equation $x + 2y - 3z = 0$, as shown in Fig. 8. A′ is the point (2, 4, 1), and M is the midpoint of AA′.

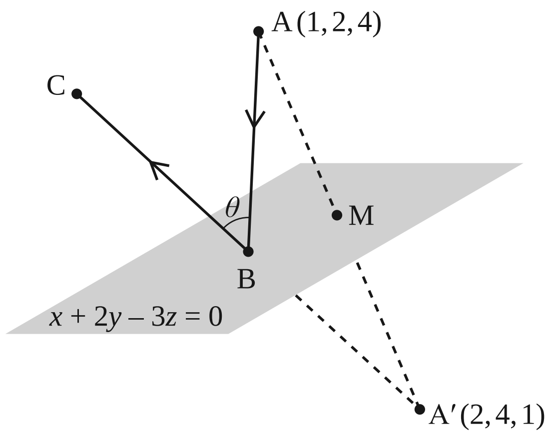

Fig. 8

(i) Show that AA′ is perpendicular to the plane $x + 2y - 3z = 0$, and that M lies in the plane. **[4]**

The vector equation of the line AB is $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

(ii) Find the coordinates of B, and a vector equation of the line A′B. **[6]**

(iii) Given that A′BC is a straight line, find the angle θ. **[4]**

(iv) Find the coordinates of the point where BC crosses the Oxz plane (the plane containing the x- and z-axes) **[3]**

2 A piece of cloth ABDC is attached to the tops of vertical poles AE, BF, DG and CH, where E, F, G and H are at ground level (see Fig. 7). Coordinates are as shown, with lengths in metres. The length of pole DG is k metres.

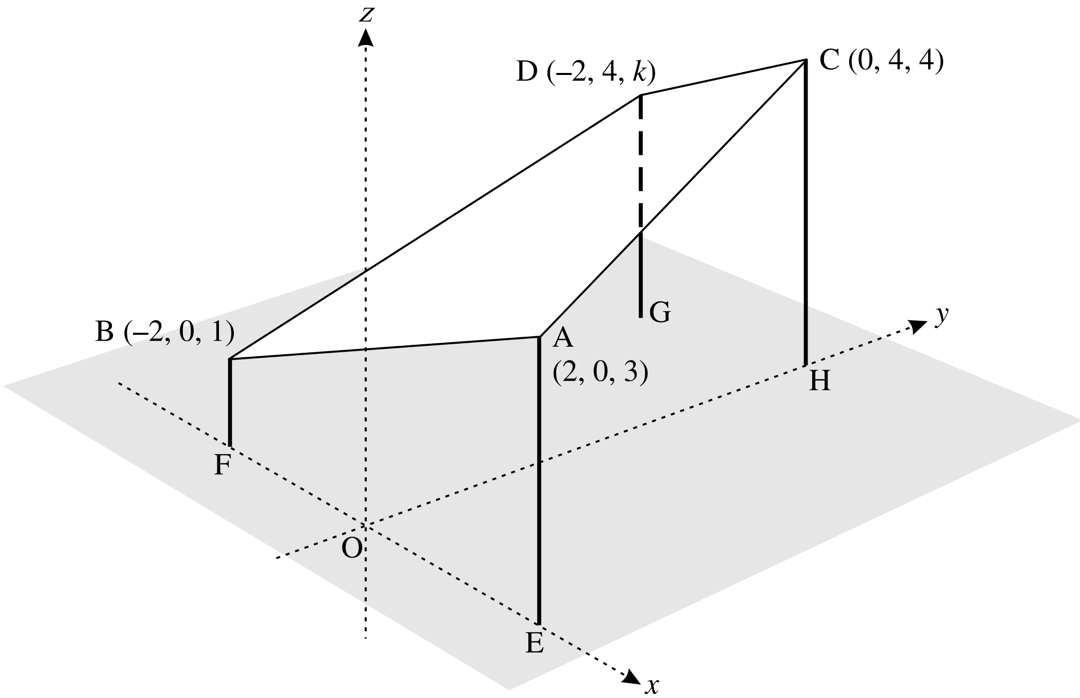

Fig. 7

(i) Write down the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Hence calculate the angle BAC. **[6]**

(ii) Verify that the equation of the plane ABC is $x + y - 2z + d = 0$, where d is a constant to be determined.

Calculate the acute angle the plane makes with the horizontal plane. **[7]**

(iii) Given that A, B, D and C are coplanar, show that $k = 3$.

Hence show that ABDC is a trapezium, and find the ratio of CD to AB. **[5]**

3 A straight pipeline AB passes through a mountain. With respect to axes Oxyz, with Ox due East, Oy due North and Oz vertically upwards, A has coordinates (−200, 100, 0) and B has coordinates (100, 200, 100), where units are metres.

(i) Verify that $\overrightarrow{AB} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ and find the length of the pipeline. **[3]**

(ii) Write down a vector equation of the line AB, and calculate the angle it makes with the vertical. **[6]**

A thin flat layer of hard rock runs through the mountain. The equation of the plane containing this layer is $x + 2y + 3z = 320$.

(iii) Find the coordinates of the point where the pipeline meets the layer of rock. **[4]**

(iv) By calculating the angle between the line AB and the normal to the plane of the layer, find the angle at which the pipeline cuts through the layer. **[5]**

4 When a light ray passes from air to glass, it is deflected through an angle. The light ray ABC starts at point A (1, 2, 2), and enters a glass object at point B (0, 0, 2). The surface of the glass object is a plane with normal vector **n**. Fig. 7 shows a cross-section of the glass object in the plane of the light ray and **n**.

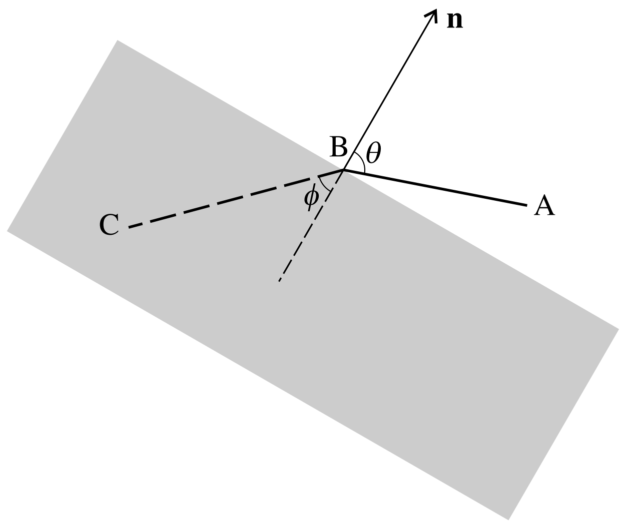

Fig. 7

(i) Find the vector $\overrightarrow{AB}$ and a vector equation of the line AB. **[2]**

The surface of the glass object is a plane with equation $x + z = 2$. AB makes an acute angle θ with the normal to this plane.

(ii) Write down the normal vector **n**, and hence calculate θ, giving your answer in degrees. **[5]**

The line BC has vector equation $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}$. This line makes an acute angle ϕ with the normal to the plane.

(iii) Show that $\phi = 45°$. **[3]**

(iv) Snell's Law states that $\sin\theta = k \sin\phi$, where k is a constant called the refractive index. Find k. **[2]**

The light ray leaves the glass object through a plane with equation $x + z = -1$. Units are centimetres.

(v) Find the point of intersection of the line BC with the plane $x + z = -1$. Hence find the distance the light ray travels through the glass object. **[5]**