1 The upper and lower surfaces of a coal seam are modelled as planes ABC and DEF, as shown in Fig. 8. All dimensions are metres.

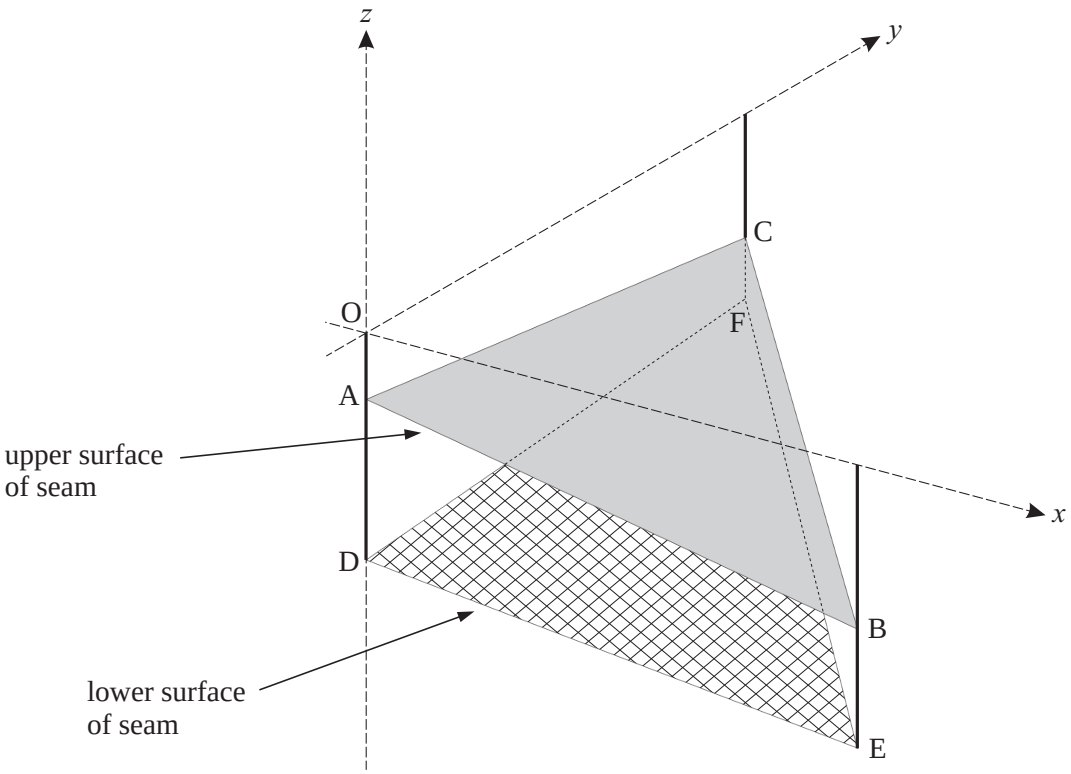

Fig. 8

Relative to axes O*x* (due east), O*y* (due north) and O*z* (vertically upwards), the coordinates of the points are as follows.

A: (0, 0, −15) B: (100, 0, −30) C: (0, 100, −25)
D: (0, 0, −40) E: (100, 0, −50) F: (0, 100, −35)

(i) Verify that the cartesian equation of the plane ABC is $3x + 2y + 20z + 300 = 0$. [3]

(ii) Find the vectors $\overrightarrow{DE}$ and $\overrightarrow{DF}$. Show that the vector $2\mathbf{i} - \mathbf{j} + 20\mathbf{k}$ is perpendicular to each of these vectors. Hence find the cartesian equation of the plane DEF. [6]

(iii) By calculating the angle between their normal vectors, find the angle between the planes ABC and DEF. [4]

It is decided to drill down to the seam from a point R (15, 34, 0) in a line perpendicular to the upper surface of the seam. This line meets the plane ABC at the point S.

(iv) Write down a vector equation of the line RS.

Calculate the coordinates of S. [5]

2 Write down normal vectors to the planes $2x + 3y + 4z = 10$ and $x - 2y + z = 5$.

Hence show that these planes are perpendicular to each other. [4]

3 Verify that the point $(-1, 6, 5)$ lies on both the lines

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}.$$

Find the acute angle between the lines. [7]

4 A computer-controlled machine can be programmed to make cuts by entering the equation of the plane of the cut, and to drill boles by entering the equation of the line of the hole.

A 20 cm x 30 cm x 30 cm cuboid is to be cut and drilled. The cuboid is positioned relative to *x*-, *y*- and z-axes as shown in Fig. 8.1.

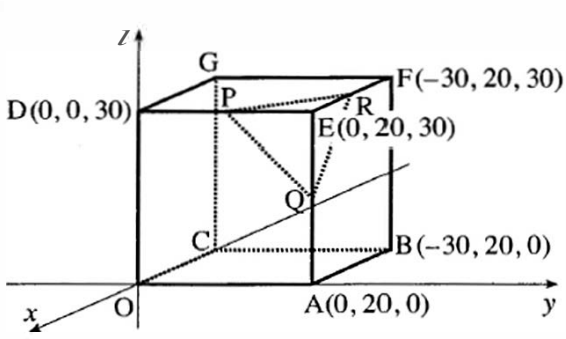

Fig. 8.1

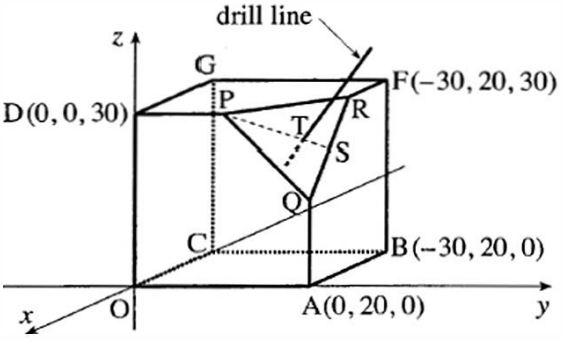

Fig. 8.2

First, a plane cut is made to remove the comer at E. The cut goes through the points P, Q and R, which are the midpoints of the sides ED, EA and EF respectively.

(i) Write down the coordinates of P, Q and R.

Hence show that $\overrightarrow{PQ} = \begin{pmatrix} 0 \\ \_ \\ \_ \end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix} \_ \\ 1 \\ -15 \end{pmatrix}$. [4]

(ii) Show that the vector $\begin{pmatrix} 15 \\ \_ \\ \_ \end{pmatrix}$ is perpendicular to the plane through P, Q and R.

Hence find the cartesian equation of this plane. (5)

A hole is then drilled perpendicular to triangle PQR, as shown in Fig. 8.2. The hole passes through the triangle at the point T which divides the line PS in the ratio 2: 1, where S is the midpoint of QR.

(iii) Write down the coordinates of S, and show that the point T has coordinates (–5. 16, 25). [4]

(iv) Write down a vector equation of the line of the drill hole.

Hence determine whether or not this line passes through C. [5]

5 A tent has vertices ABCDEF with coordinates as shown in Fig. 7. Lengths are in metres. The O*xy* plane is horizontal.

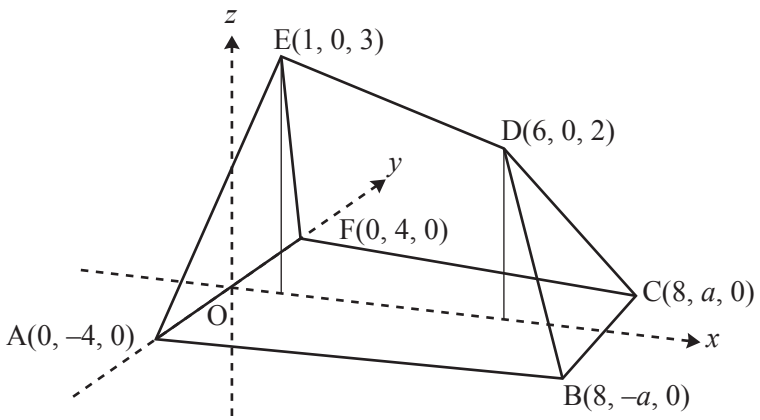

Fig. 7

(i) Find the length of the ridge of the tent DE, and the angle this makes with the horizontal. **[4]**

(ii) Show that the vector $\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ is normal to the plane through A, D and E.

Hence find the equation of this plane. Given that B lies in this plane, find a. **[7]**

(iii) Verify that the equation of the plane BCD is $x + z = 8$.

Hence find the acute angle between the planes ABDE and BCD. **[6]**