Question			Answer	Marks	Guidance
1	(i)		A: $0 + 6.(-2) + 12 = 0$ B: $3 + 6.(-2.5) + 12 = 0$ E: $0 + 6.(-2) + 12 = 0$	B2,1,0	B1 for two points verified (must see as a minimum $-12 + 12 = 0$, $3 - 15 + 12 = 0$, $-12 + 12 = 0$) or any valid complete method for either finding or verifying that $x+6y+12=0$ gets M1 A1
			At F, $2 + 6a + 12 = 0$	M1	Substitution of F into $x+6y+12=0$
			$\Rightarrow \quad 6a = -14,\ a = -14/6 = -7/3$ *	A1	www NB **AG**
				[4]	
1	(ii)	(A)	$\overrightarrow{DH}.\begin{pmatrix}1\\-6\\0\end{pmatrix}=\begin{pmatrix}1\\-6\\0\end{pmatrix}\begin{pmatrix}0\\0\\3\end{pmatrix}=1\times0+(-6)\times0+0\times3=0$	B1	scalar product with a **direction** vector in the plane (including evaluation and = 0) (**OR** M1 forms a vector product with at least two correct terms in solution)
			$\overrightarrow{DC}.\begin{pmatrix}1\\-6\\0\end{pmatrix}=\begin{pmatrix}3\\0.5\\0\end{pmatrix}.\begin{pmatrix}1\\-6\\0\end{pmatrix}=3\times1+0.5\times(-6)+0\times0=0$	B1	scalar product with second **direction** vector, with evaluation. (following OR above, A1 correct ie a multiple of $\mathbf{i}-6\mathbf{j}$) (NB finding only one direction vector and its scalar product is B1 only)
				[2]	
1	(ii)	(B)	**r**.(**i** – 6**j**) = **j**.(**i** – 6**j**)	M1	$\mathbf{r}\cdot\mathbf{n}=\mathbf{a}\cdot\mathbf{n}$ with $\mathbf{n}=\begin{pmatrix}1\\-6\\0\end{pmatrix}$ and $\mathbf{a}=\begin{pmatrix}0\\1\\3\end{pmatrix}$ or $\begin{pmatrix}0\\1\\0\end{pmatrix}$ or $\begin{pmatrix}3\\1.5\\0\end{pmatrix}$ **or** substituting H(0, 1, 3) or D(0, 1, 0) or C(3, 1.5, 0) into $x-6y=d$
			$\Rightarrow x - 6y + 6 = 0$	A1	oe (isw if d found correctly and $x - 6y = d$ stated) B2 www correct equation stated
				[2]	
1	(ii)	(C)	$2 - 6b + 6 = 0 \Rightarrow b = 4/3$	B1	oe – exact answer
			$\text{FG} = 1\frac{1}{3}+2\frac{1}{3}=3\frac{2}{3}$	B1	oe – exact answer
				[2]	

Question			Answer	Marks	Guidance
1	(iii)		($\overrightarrow{FE}$ =) $-2\mathbf{i} + (1/3)\mathbf{j} + \mathbf{k}$, ($\overrightarrow{FB}$ =) $\mathbf{i} - (1/6)\mathbf{j} - 2\mathbf{k}$	B1 B1	or $(\overrightarrow{EF} =) 2\mathbf{i} + (-1/3)\mathbf{j} - \mathbf{k}$ or $(\overrightarrow{BF} =) -\mathbf{i} + (1/6)\mathbf{j} + 2\mathbf{k}$
			$\cos\theta = \dfrac{-2(1)+(1/3)(-1/6)+1(-2)}{\sqrt{4+1/9+1}\sqrt{1+1/36+4}}$	M1	$\cos\theta = \left(\overrightarrow{FE}\cdot\overrightarrow{FB}\right)/\left(\lvert\overrightarrow{FE}\rvert\lvert\overrightarrow{FB}\rvert\right)$ (oe) follow through their FE and FB (allow any combination of FE, EF with FB, BF) – allow one sign slip only
			$\theta = \arccos\left(\dfrac{-\dfrac{73}{18}}{\dfrac{\sqrt{46}}{3}\times\dfrac{\sqrt{181}}{6}}\right)$	A1	$\arccos\left(\dfrac{-2-1/18-2}{5.069}\right) = \arccos(\pm -0.800)$
			$\Rightarrow \quad q = 143°$	A1 **[5]**	3sf or better (or 2.5(0) radians or better). Allow candidates who find the acute angle using either $\overrightarrow{EF}$ with $\overrightarrow{FB}$ or $\overrightarrow{FE}$ with $\overrightarrow{BF}$ and then state the obtuse angle. **Do not isw those who find the obtuse angle and then state the acute angle.** Note: $90 + 2\arctan(1/2)$ is 0/5
		OR	EF = $\sqrt{46}/3$, FB = $\sqrt{181}/6$, EB = $\sqrt{73}/2$	B3,2,1,0	One mark for each (2.26, 2.24, 4.27)
			$\theta = \arccos\left(\dfrac{\left(\sqrt{46}/3\right)^2 + \left(\sqrt{181}/6\right)^2 - \left(\sqrt{73}/2\right)^2}{2(\sqrt{46}/3)(\sqrt{181}/6)}\right)$	M1 A1	cosine rule correct with their EF, FB, EB $q = 143°$
1	(iv)		z coordinate of P is 5/2	B1	stating the correct z-coordinate of P; ignore incorrect x and y coordinates (or stated in a position vector)
			$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} = \begin{pmatrix}1\\-13/6\\5/2\end{pmatrix} + \dfrac{1}{3}\left(\begin{pmatrix}0\\1\\3\end{pmatrix} - \begin{pmatrix}1\\-13/6\\5/2\end{pmatrix}\right)$	M1	Complete method for finding the z-coordinate of Q or $\overrightarrow{OQ} = (\overrightarrow{OH}) + (2/3)(\overrightarrow{HP})$ or $\overrightarrow{OQ} = (2/3)(\overrightarrow{OP}) + (1/3)(\overrightarrow{OH})$
			so height of Q is 8/3 (metres above ground)	A1 **[3]**	2.67 or better

Question			Answer	Marks	Guidance
2	(i)		$AB = \sqrt{5^2 + (-2)^2} = \sqrt{29}$	B1	5.39 or better (condone sign error in vector for B1)
			$AC = \sqrt{3^2 + 4^2} = 5$	B1	Accept √25 (condone sign error in vector for B1)
			$\cos\theta = \dfrac{\begin{pmatrix}5\\0\\-2\end{pmatrix}.\begin{pmatrix}3\\4\\0\end{pmatrix}}{\sqrt{29}.5} = \dfrac{15+0+0}{5\sqrt{29}} = 0.5571$	M1	$\cos\theta = \dfrac{\text{scalar product of AB with AC}}{\|AB\|.\|AC\|}$ (accept BA/CA) **with substitution** condone a single numerical error provided method is clearly understood **[OR** Cosine Rule, as far as cos θ= correct numerical expression]
				A1	**www** ± 0.5571, 0.557,15/5√29,15/√25√29 oe or better soi (± for method only)
			$\Rightarrow \quad \theta = 56.15°$	A1	www Accept answers that round to 56.1º or 56.2º or 0.98 radians (or better) **NB vector 5i+0j+2k leads to apparently correct answer but loses all A marks in part(i)**
			Area = ½ × 5 × $\sqrt{29}$ × sin θ	M1	Using their AB,AC, ∠CAB. Accept any valid method using trigonometry
			= 11.18	A1	Accept 5√5 and answers that round to 11.18 or 11.19 (2dp) **www** or SCA1 for accurate work soi rounded at the last stage to 11.2 (but not from an incorrect answer, say from an incorrect angle or from say 11.17 or 11.22 stated and rounded to 11.2) We will not accept inaccurate work from over rounded answers for the final mark.
				[7]	

Question			Answer	Marks	Guidance
2	**(ii)**	(*A*)	$\overrightarrow{AB}.\begin{pmatrix}4\\-3\\10\end{pmatrix}=\begin{pmatrix}5\\0\\-2\end{pmatrix}\begin{pmatrix}4\\-3\\10\end{pmatrix}=5.4+0.(-3)+(-2).10=0$ $\overrightarrow{AC}.\begin{pmatrix}4\\-3\\10\end{pmatrix}=\begin{pmatrix}3\\4\\0\end{pmatrix}.\begin{pmatrix}4\\-3\\10\end{pmatrix}=3\times4+4\times(-3)+0\times10=0$	B1 B1 **[2]**	Scalar product with one vector in the plane with numerical expansion shown. Scalar product, as above, with evaluation, with a second vector. **NB vectors are not unique** SCB2 finding the equation of plane first by any valid method (or using vector product) and then clearly stating that the normal is proportional to the coefficients. SC For candidates who substitute all three points in the plane $4x$-$3y$+$10z = c$ and show that they give the same result, award M1 If they include a statement explaining why this means that 4**i**-3**j**+10**k** is normal they can gain A1.
2	**(ii)**	(*B*)	$4x-3y+10z=c$ $\Rightarrow 4x-3y+10z+12=0$	M1 A1 **[2]**	Required form **and** substituting the co-ordinates of a point on the plane oe If found in (A) it must be clearly referred to in (B) to gain the marks. Do not accept vector equation of the plane, as ‘Hence’. 4**i-3j**+10**k** = – 12 is M1A0

Question			Answer	Marks	Guidance
2	**(iii)**		$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}$.	B1	Need $\mathbf{r} =$ (or $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$)
			$+\lambda \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix}$	B1	oe
			Meets $4x - 3y + 10z + 12 = 0$ when	M1	Subst their 4λ, $4 - 3\lambda$, $5+10\lambda$ in equation of their plane from (ii)
			$16\lambda - 3(4 - 3\lambda) + 10(5 + 10\lambda) + 12 = 0$		
			$\Rightarrow \quad 125\lambda = -50$, $\lambda = -0.4$	A1	$\lambda = -0.4$ (NB not unique)
			So meets plane ABC at $(-1.6, 5.2, 1)$	A1	cao www (condone vector)
				[5]	
2	**(iv)**		height $= \sqrt{(1.6^2 + (-1.2)^2 + 4^2)} = \sqrt{20}$	B1ft	ft their (iii)
			volume $= 11.18 \times \sqrt{20} / 3 = 16.7$	B1cao	50/3 or answers that round to 16.7 www and not from incorrect answers from (iii) ie not from say (1.6,2.8,9)
				[2]	

Question			Answer	Marks	Guidance
3	**(i)**		$\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \ldots$	B1	need **r (**or another letter**)** = or $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ for first B1
			$\ldots + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$	B1	
					NB answer is not unique eg $\mathbf{r} = \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$
					Accept **i/j/k** form and condone row vectors.
				[2]	
3	**(ii)**		$x + 3y + 2z = 4$		
			$\Rightarrow \quad -2\lambda + 3(1+\lambda) + 2(3+2\lambda) = 4$	M1	substituting their line in plane equation (condone a slip if intention clear)
			$\Rightarrow \quad 5\lambda = -5, \ \lambda = -1$	A1	www cao **NB λ is not unique** as depends on choice of line in (i)
			so point of intersection is (2, 0, 1)	A1	www cao
				[3]	
3	**(iii)**		Angle between –2**i + j** + 2**k** and **i** + 3**j** +2**k** is θ where	M1	Angl between **i**+3**j**+2**k** and **their** direction from (i) ft condone a single sign slip if intention clear
			$\cos\theta = \dfrac{-2\times1+1\times3+2\times2}{\sqrt{9}\sqrt{14}} = \dfrac{5}{3\sqrt{14}}$	M1	correct formula (including cosine) , with substitution, for these vectors condone a single numerical or sign slip if intention is clear
			$\Rightarrow \quad \theta = 63.5°$	A1	www cao (63.5 in degrees (or better) or 1.109 radians or better)
				[3]	