1 A glass ornament OABCDEFG is a truncated pyramid on a rectangular base (see Fig. 7). All dimensions are in centimetres.

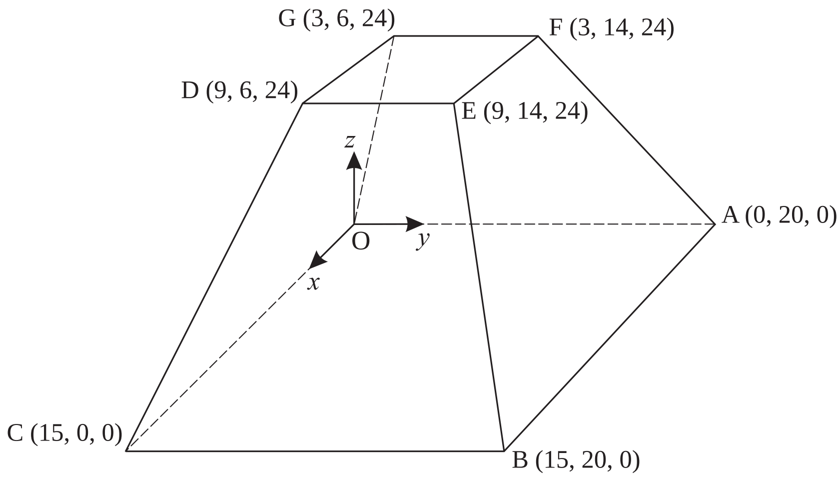

Fig. 7

(i) Write down the vectors $\overrightarrow{CD}$ and $\overrightarrow{CB}$. [2]

(ii) Find the length of the edge CD. [2]

(iii) Show that the vector $4\mathbf{i} + \mathbf{k}$ is perpendicular to the vectors $\overrightarrow{CD}$ and $\overrightarrow{CB}$. Hence find the cartesian equation of the plane BCDE. [5]

(iv) Write down vector equations for the lines OG and AF.

Show that they meet at the point P with coordinates $(5, 10, 40)$. [5]

You may assume that the lines CD and BE also meet at the point P.

The volume of a pyramid is $\frac{1}{3} \times$ area of base $\times$ height.

(v) Find the volumes of the pyramids POABC and PDEFG.

Hence find the volume of the ornament. [4]

2 In a game of rugby, a kick is to be taken from a point P (see Fig. 7). P is a perpendicular distance y metres from the line TOA. Other distances and angles are as shown.

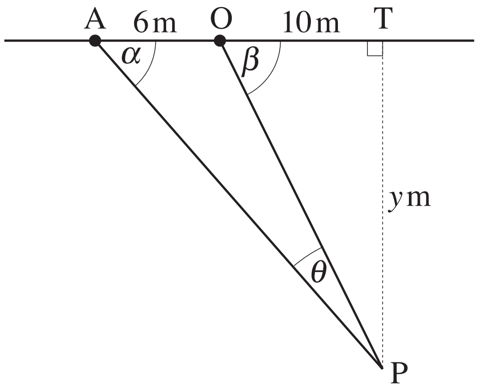

Fig. 7

(i) Show that $\theta = \beta - \alpha$, and hence that $\tan\theta = \dfrac{6y}{160 + y^2}$.

Calculate the angle θ when $y = 6$. [8]

(ii) By differentiating implicitly, show that $\dfrac{d\theta}{dy} = \dfrac{6(160 - y^2)}{(160 + y^2)^2}\cos^2\theta$. [5]

(iii) Use this result to find the value of y that maximises the angle θ. Calculate this maximum value of θ. [You need not verify that this value is indeed a maximum.] [4]

3 Fig. 6 shows a lean-to greenhouse ABCDHEFG. With respect to coordinate axes Oxyz, the coordinates of the vertices are as shown. All distances are in metres. Ground level is the plane $z = 0$.

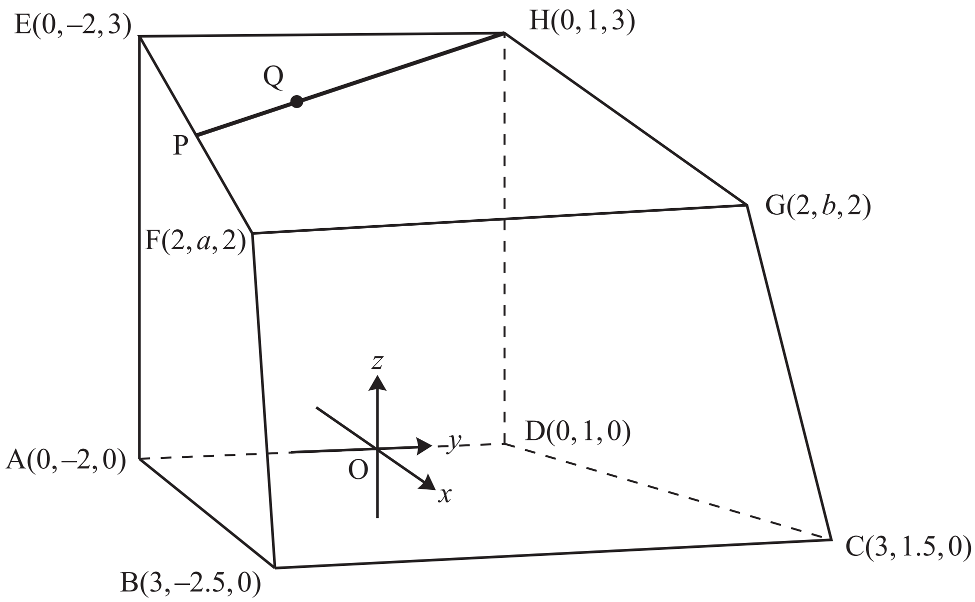

Fig. 6

(i) Verify that the equation of the plane through A, B and E is $x + 6y + 12 = 0$.

Hence, given that F lies in this plane, show that $a = -2\frac{1}{3}$. **[4]**

(ii) (A) Show that the vector $\begin{pmatrix} 1 \\ -6 \\ 0 \end{pmatrix}$ is normal to the plane DHC. **[2]**

(B) Hence find the cartesian equation of this plane. **[2]**

(C) Given that G lies in the plane DHC, find b and the length FG. **[2]**

(iii) Find the angle EFB. **[5]**

A straight wire joins point H to a point P which is half way between E and F. Q is a point two-thirds of the way along this wire, so that HQ = 2QP.

(iv) Find the height of Q above the ground. **[3]**

4 A computer-controlled machine can be programmed to make cuts by entering the equation of the plane of the cut, and to drill holes by entering the equation of the line of the hole.

A 20 cm x 30 cm x 30 cm cuboid is to be cut and drilled. The cuboid is positioned relative to x-, y- and z-axes as shown in Fig. 8.1.

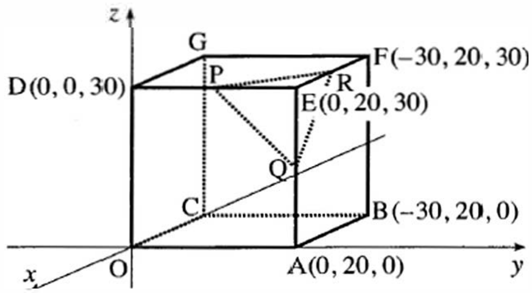

Fig.8.1

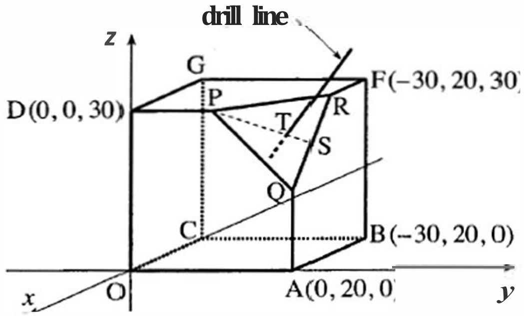

Fig. 8.2

Fkst, a plane cut is made to remove the comer at E. The cut goes through the points P. Q and R, which are the midpoints of the sides ED, EA and EF respectively.

(i) Write down the coordinates of P, Q and R.

Henceshowlhat Ni{:i) and {:n [4]

(;;) Show that Lhe veeto, G) is pc,pcndicula, to the plone through P, Q ,nd R

Hence find the Cartesian equation of this plane. [5)

A hole is then drilled perpendicular to lriangle PQR, as shown in Fig. 82. The hole passes through the triangle at the point T which divides the line PS in the ratio 2: I, where S is the midpoint of QR.

(iii) Write down the coordinates of S, and show that the point T has coordinates (-5, 16. 25). [4]

(iv) Write down a vector equation of the line of the drill hole.

Hence determine whether or not this line passes through C. (5J