1(i) $\overrightarrow{CD}=\begin{pmatrix}-6\\6\\24\end{pmatrix}$ $\overrightarrow{CB}=\begin{pmatrix}0\\20\\0\end{pmatrix}$.	B1 B1 [2]	
(ii) $\sqrt{(-6)^2+6^2+24^2}$ = 25.46 cm	M1 A1 [2]	
(iii) $\overrightarrow{CD}.\begin{pmatrix}4\\0\\1\end{pmatrix}=\begin{pmatrix}-6\\6\\24\end{pmatrix}.\begin{pmatrix}4\\0\\1\end{pmatrix}=-24+0+24=0$ $\overrightarrow{CB}.\begin{pmatrix}4\\0\\1\end{pmatrix}=\begin{pmatrix}0\\20\\0\end{pmatrix}.\begin{pmatrix}4\\0\\1\end{pmatrix}=0+0+0=0$ $\Rightarrow$ plane BCDE is $4x + z = c$ At C (say) $4\times15 + 0 = c \Rightarrow c = 60$ $\Rightarrow$ plane BCDE is $4x + z = 60$	M1 A1 B1 M1 A1 [5]	using scalar product or other equivalent methods
(iv) OG: $\mathbf{r}=\begin{pmatrix}0\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}3\\6\\24\end{pmatrix}$ AF: $\mathbf{r}=\begin{pmatrix}0\\20\\0\end{pmatrix}+\mu\begin{pmatrix}3\\-6\\24\end{pmatrix}$ At (5, 10, 40) , $3\lambda = 5 \Rightarrow \lambda = 5/3$ $\Rightarrow 6\lambda = 10$, $24\lambda = 40$, so consistent. At (5, 10, 40) , $3\mu = 5 \Rightarrow \mu = 5/3$ $\Rightarrow 20 - 6\mu = 10$, $24\mu = 40$, so consistent. So lines meet at (5, 10, 40)*	B1 B1 M1 E1 E1 [5]	 evaluating parameter and checking consistency. [or other methods, e.g. solving]
(v) h=40 POABC: $V = 1/3 \times 20 \times 15 \times 40$ $= 4000$ cm^3. PDEFG: $V = 1/3 \times 8 \times 6 \times (40-24)$ $= 256$ cm^3 $\Rightarrow$ vol of ornament = $4000 - 256 = 3744$ cm^3	B1 M1 A1 A1 [4]	soi 1/3 x w x d x h used for either –condone one error both volumes correct cao

2(i) $A\hat{O}P = 180 - \beta = 180 - \alpha - \theta$ $\Rightarrow \beta = \alpha + \theta$ $\Rightarrow \theta = \beta - \alpha$ $\tan\theta = \tan(\beta - \alpha)$ $= \dfrac{\tan\beta - \tan\alpha}{1 + \tan\beta\tan\alpha}$ $= \dfrac{\frac{y}{10} - \frac{y}{16}}{1 + \frac{y}{10}.\frac{y}{16}}$ $= \dfrac{16y - 10y}{160 + y^2}$ $= \dfrac{6y}{160 + y^2}$ * When y = 6, $\tan\theta$ = 36/196 $\Rightarrow \theta = 10.4°$	M1 M1 E1 M1 A1 E1 M1 A1 cao [8]	Use of sum of angles in triangle OPT and AOP oe SC B1 for $\beta=\alpha+\theta$, $\theta=\beta-\alpha$ no justification Use of Compound angle formula Substituting values for tan α and tan β www accept radians
(ii) $\sec^2\theta\dfrac{d\theta}{dy} = \dfrac{(160+y^2)6 - 6y.2y}{(160+y^2)^2}$ $= \dfrac{6(160+y^2-2y^2)}{(160+y^2)^2}$ $\Rightarrow \dfrac{d\theta}{dy} = \dfrac{6(160-y^2)}{(160+y^2)^2}\cos^2\theta$ *	M1 M1 A1 A1 E1 [5]	$\sec^2\theta\dfrac{d\theta}{dy} = \ldots$ quotient rule correct expression simplifying numerator www
(iii) $d\theta/dy = 0$ when $160 - y^2 = 0$ $\Rightarrow y^2 = 160$ $\Rightarrow y = 12.65$ When y = 12.65, $\tan\theta$ = 0.237… $\Rightarrow \theta = 13.3°$	M1 A1 M1 A1cao [4]	oe accept radians

Question			Answer	Marks	Guidance
3	(i)		A: 0 + 6.(−2) + 12 = 0 B: 3 + 6.(−2.5) + 12 = 0 E: 0 + 6.(−2) + 12 = 0 At F, 2 + 6*a* + 12 = 0 ⇒ 6*a* = −14, *a* = −14/6 = −7/3 *	B2,1,0 M1 A1 **[4]**	B1 for two points verified (must see as a minimum –12 + 12 = 0, 3 – 15 + 12 = 0, –12 + 12 = 0) or any valid complete method for either finding or verifying that $x+6y+12=0$ gets M1 A1 Substitution of F into $x+6y+12=0$ www NB **AG**
3	(ii)	(A)	$\overrightarrow{DH}.\begin{pmatrix}1\\-6\\0\end{pmatrix}=\begin{pmatrix}1\\-6\\0\end{pmatrix}\begin{pmatrix}0\\0\\3\end{pmatrix}=1\times0+(-6)\times0+0\times3=0$ $\overrightarrow{DC}.\begin{pmatrix}1\\-6\\0\end{pmatrix}=\begin{pmatrix}3\\0.5\\0\end{pmatrix}.\begin{pmatrix}1\\-6\\0\end{pmatrix}=3\times1+0.5\times(-6)+0\times0=0$	B1 B1 **[2]**	scalar product with a **direction** vector in the plane (including evaluation and = 0) (**OR** M1 forms a vector product with at least two correct terms in solution) scalar product with second **direction** vector, with evaluation. (following OR above, A1 correct ie a multiple of $\mathbf{i}-6\mathbf{j}$) (NB finding only one direction vector and its scalar product is B1 only)
3	(ii)	(B)	**r**.(**i** – 6**j**) = **j**.(**i** – 6**j**) ⇒ $x - 6y + 6 = 0$	M1 A1 **[2]**	$\mathbf{r}\cdot\mathbf{n}=\mathbf{a}\cdot\mathbf{n}$ with $\mathbf{n}=\begin{pmatrix}1\\-6\\0\end{pmatrix}$ and $\mathbf{a}=\begin{pmatrix}0\\1\\3\end{pmatrix}$ or $\begin{pmatrix}0\\1\\0\end{pmatrix}$ or $\begin{pmatrix}3\\1.5\\0\end{pmatrix}$ **or** substituting H(0, 1, 3) or D(0, 1, 0) or C(3, 1.5, 0) into $x-6y=d$ oe (isw if *d* found correctly and *x* – 6y = *d* stated) B2 www correct equation stated
3	(ii)	(C)	2 −6b + 6 = 0 ⇒ b = 4/3 $\text{FG}=1\frac{1}{3}+2\frac{1}{3}=3\frac{2}{3}$	B1 B1 **[2]**	oe – exact answer oe – exact answer

Question			Answer	Marks	Guidance
3	(iii)		$(\overrightarrow{FE}=)\ -2\mathbf{i}+(1/3)\mathbf{j}+\mathbf{k}$, $(\overrightarrow{FB}=)\ \mathbf{i}-(1/6)\mathbf{j}-2\mathbf{k}$	B1 B1	or $(\overrightarrow{EF}=)2\mathbf{i}+(-1/3)\mathbf{j}-\mathbf{k}$ or $(\overrightarrow{BF}=)-\mathbf{i}+(1/6)\mathbf{j}+2\mathbf{k}$
			$\cos\theta=\dfrac{-2(1)+(1/3)(-1/6)+1(-2)}{\sqrt{4+1/9+1}\sqrt{1+1/36+4}}$	M1	$\cos\theta=\left(\overrightarrow{FE}\cdot\overrightarrow{FB}\right)/\left(\lvert\overrightarrow{FE}\rvert\lvert\overrightarrow{FB}\rvert\right)$ (oe) follow through their FE and FB (allow any combination of FE, EF with FB, BF) – allow one sign slip only
			$\theta=\arccos\left(\dfrac{-\dfrac{73}{18}}{\dfrac{\sqrt{46}}{3}\times\dfrac{\sqrt{181}}{6}}\right)$	A1	$\arccos\left(\dfrac{-2-1/18-2}{5.069}\right)=\arccos(\pm-0.800)$
			$\Rightarrow\quad q=143°$	A1 **[5]**	3sf or better (or 2.5(0) radians or better). Allow candidates who find the acute angle using either $\overrightarrow{EF}$ with $\overrightarrow{FB}$ or $\overrightarrow{FE}$ with $\overrightarrow{BF}$ and then state the obtuse angle. **Do not isw those who find the obtuse angle and then state the acute angle.** Note: $90+2\arctan(1/2)$ is 0/5
		OR	EF = $\sqrt{46}/3$, FB = $\sqrt{181}/6$, EB = $\sqrt{73}/2$	B3,2,1,0	One mark for each (2.26, 2.24, 4.27)
			$\theta=\arccos\left(\dfrac{(\sqrt{46}/3)^2+(\sqrt{181}/6)^2-(\sqrt{73}/2)^2}{2(\sqrt{46}/3)(\sqrt{181}/6)}\right)$	M1 A1	cosine rule correct with their EF, FB, EB q = 143°
3	(iv)		z coordinate of P is 5/2	B1	stating the correct z-coordinate of P; ignore incorrect x and y coordinates (or stated in a position vector)
			$\overrightarrow{OQ}=\overrightarrow{OP}+\overrightarrow{PQ}=\begin{pmatrix}1\\-13/6\\5/2\end{pmatrix}+\dfrac{1}{3}\left(\begin{pmatrix}0\\1\\3\end{pmatrix}-\begin{pmatrix}1\\-13/6\\5/2\end{pmatrix}\right)$	M1	Complete method for finding the z-coordinate of Q or $\overrightarrow{OQ}=(\overrightarrow{OH})+(2/3)(\overrightarrow{HP})$ or $\overrightarrow{OQ}=(2/3)(\overrightarrow{OP})+(1/3)(\overrightarrow{OH})$
			so height of Q is 8/3 (metres above ground)	A1 **[3]**	2.67 or better

4 (i) P is (0, 10, 30) Q is (0, 20, 15) R is (–15, 20, 30) $\Rightarrow \overrightarrow{PQ} = \begin{pmatrix} 0-0 \\ 20-10 \\ 15-30 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ -15 \end{pmatrix}$ * $\Rightarrow \overrightarrow{PR} = \begin{pmatrix} -15-0 \\ 20-10 \\ 30-30 \end{pmatrix} = \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix}$ *	**B2,1,0** **E1** **E1** **[4]**	

(ii) $\begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}0\\10\\-15\end{pmatrix}=0+30-30=0$ $\begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}-15\\10\\0\end{pmatrix}=-30+30+0=0$ $\Rightarrow \begin{pmatrix}2\\3\\2\end{pmatrix}$ is normal to the plane $\Rightarrow$ equation of plane is $2x + 3y + 2z = c$ At P (say), $x = 0$, $y = 10$, $z = 30$ $\Rightarrow c = 2\times0 + 3\times10 + 2\times30 = 90$ $\Rightarrow$ equation of plane is $2x + 3y + 2z = 90$	**M1** **E1** **M1** **M1dep** **A1 cao** **[5]**	Scalar product with 1 vector in the plane OR vector x product oe $2x + 3y + 2z = c$ **or** an appropriate vector form substituting to find c **or** completely eliminating parameters
(iii) S is $(-7\frac{1}{2}, 20, 22\frac{1}{2})$ $\overrightarrow{OT}=\overrightarrow{OP}+\frac{2}{3}\overrightarrow{PS}$ $=\begin{pmatrix}0\\10\\30\end{pmatrix}+\frac{2}{3}\begin{pmatrix}-7\frac{1}{2}\\10\\-7\frac{1}{2}\end{pmatrix}=\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}$ So T is $(-5, 16\frac{2}{3}, 25)$ *	**B1** **M1** **A1ft** **E1** **[4]**	**Or** $\frac{1}{3}(\overrightarrow{OP}+\overrightarrow{OR}+\overrightarrow{OQ})$ oe ft their S **Or** $\frac{1}{3}\begin{pmatrix}0\\10\\30\end{pmatrix}+\frac{2}{3}\begin{pmatrix}-7\frac{1}{2}\\20\\22\frac{1}{2}\end{pmatrix}$ ft their S
(iv) $\mathbf{r}=\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$ At C (–30, 0, 0): $-5 + 2\lambda = -30$, $16\frac{2}{3}+3\lambda=0$, $25 + 2\lambda = 0$ 1st and 3rd eqns give $\lambda = -12\frac{1}{2}$, not compatible with 2nd. So line does not pass through C.	**B1,B1** **M1** **A1** **E1** **[5]**	$\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}+\ldots \ldots+\lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$ Substituting coordinates of C into vector equation At least 2 relevant correct equations for λ oe www