1 A curve has parametric equations $x = \sec\theta,\ y = 2\tan\theta$.

(i) Given that the derivative of $\sec\theta$ is $\sec\theta\tan\theta$, show that $\frac{dy}{dx} = 2\,\text{cosec}\,\theta$. **[3]**

(ii) Verify that the cartesian equation of the curve is $y^2 = 4x^2 - 4$. **[2]**

Fig. 5 shows the region enclosed by the curve and the line $x = 2$. This region is rotated through 180° about the x-axis.

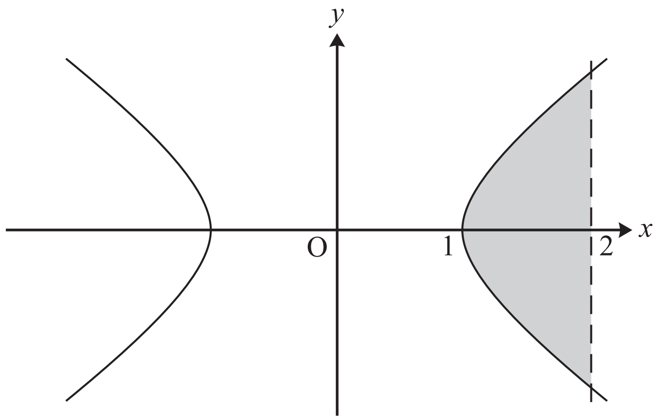

Fig. 5

(iii) Find the volume of revolution produced, giving your answer in exact form. **[3]**

2 Show that the equation $\text{cosec}\,x + 5\cot x = 3\sin x$ may be rearranged as

$$3\cos^2 x + 5\cos x - 2 = 0.$$

Hence solve the equation for $0° \leqslant x \leqslant 360°$, giving your answers to 1 decimal place. **[7]**

3 Using appropriate right-angled triangles, show that $\tan 45° = 1$ and $\tan 30° = \frac{1}{\sqrt{3}}$.

Hence show that $\tan 75° = 2 + \sqrt{3}$. **[7]**

4 Prove that $\sec^2\theta + \text{cosec}^2\theta = \sec^2\theta\ \text{cosec}^2\theta$. **[4]**

5 Solve the equation $\text{cosec}^2\theta = 1 + 2\cot\theta$, for $-180^\circ \leqslant \theta \leqslant 180^\circ$. **[6]**

6 Given that $\text{cosec}^2\theta - \cot\theta = 3$, show that $\cot^2\theta - \cot\theta - 2 = 0$.

Hence solve the equation $\text{cosec}^2\theta - \cot\theta = 3$ for $0^\circ \leqslant \theta \leqslant 180^\circ$. **[6]**

7 Given that $x = 2\sec\theta$ and $y = 3\tan\theta$, show that $\frac{x^2}{4} - \frac{y^2}{9} = 1$. [3]

8 Solve the equation

$$\sec^2\theta = 4, \quad 0 \leqslant \theta \leqslant \pi,$$

giving your answers in terms of π. [4]