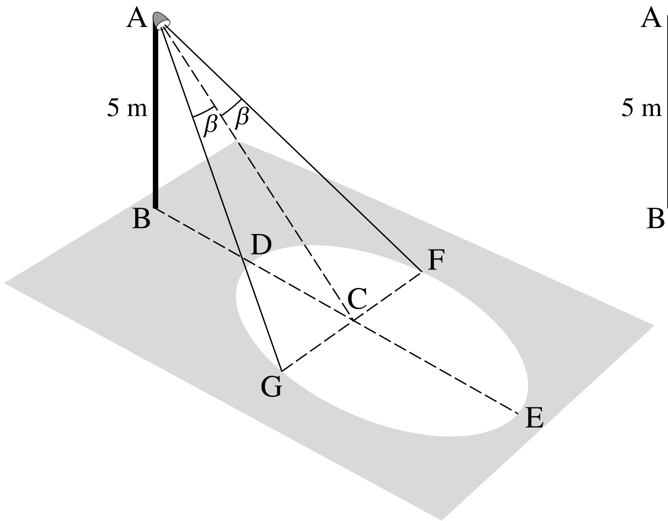
1 Solve the equation $2\sec^2\theta = 5\tan\theta$, for $0 \leqslant \theta \leqslant \pi$. **[6]**

2 Solve, correct to 2 decimal places, the equation $\cot 2\theta = 3$ for $0° \leqslant \theta \leqslant 180°$. **[4]**

3 Fig. 8 shows a searchlight, mounted at a point A, 5 metres above level ground. Its beam is in the shape of a cone with axis AC, where C is on the ground. AC is angled at α to the vertical. The beam produces an oval-shaped area of light on the ground, of length DE. The width of the oval at C is GF. Angles DAC, EAC, FAC and GAC are all β.



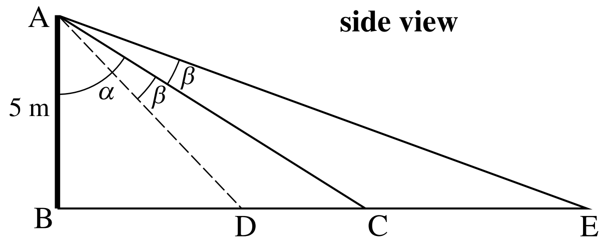


Fig. 8

In the following, all lengths are in metres.

(i) Find AC in terms of α, and hence show that GF = $10 \sec \alpha \tan \beta$. **[3]**

(ii) Show that CE = $5\big(\tan(\alpha + \beta) - \tan \alpha\big)$.

Hence show that CE = $\dfrac{5 \tan \beta \sec^2 \alpha}{1 - \tan \alpha \tan \beta}$. **[5]**

Similarly, it can be shown that CD = $\dfrac{5 \tan \beta \sec^2 \alpha}{1 + \tan \alpha \tan \beta}$. [You are **not** required to derive this result.]

You are now given that $\alpha = 45°$ and that $\tan \beta = t$.

(iii) Find CE and CD in terms of t. Hence show that DE = $\dfrac{20t}{1 - t^2}$. **[5]**

(iv) Show that GF = $10\sqrt{2}t$. **[2]**

For a certain value of β, DE = 2GF.

(v) Show that $t^2 = 1 - \dfrac{1}{\sqrt{2}}$.

Hence find this value of β. **[3]**

4 Show that $\cot 2\theta = \dfrac{1 - \tan^2 \theta}{2 \tan \theta}$.

Hence solve the equation

$$\cot 2\theta = 1 + \tan \theta \quad \text{for } 0^\circ < \theta < 360^\circ.$$ **[7]**

5 Prove that $\cot \beta - \cot \alpha = \dfrac{\sin(\alpha - \beta)}{\sin \alpha \sin \beta}$. **[3]**

6 Solve the equation $\operatorname{cosec} \theta = 3$, for $0^\circ < \theta < 360^\circ$. [3]