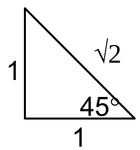
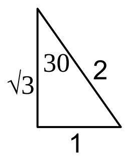
Question			Answer	Marks	Guidance
1	(i)		$x = \sec\theta,\ y = 2\tan\theta$		
			$\Rightarrow \quad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\sec^2\theta}{\sec\theta\tan\theta}$	M1A1	M1 for their $(dy/d\theta) \div \sec\theta\tan\theta$ in terms of θ A1 cao (oe) allow for unsimplified form even if subsequently cancelled incorrectly ie can isw
			$= \frac{2\sec\theta}{\tan\theta} = \frac{2}{\cos\theta}.\frac{\cos\theta}{\sin\theta} = \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta$ *	A1	cao www (NB **AG**) – must be at least one intermediate step between $\frac{2\sec\theta}{\tan\theta}$ $\frac{2}{\sin\theta}$ or $2\operatorname{cosec}\theta$
				[3]	
1	(ii)		$x^2 = \sec^2\theta = 1 + \tan^2\theta = 1 + ¼\,y^2$	M1	$\sec^2\theta = 1 + \tan^2\theta$ (oe) used
			$\Rightarrow \quad y^2 = 4(x^2 - 1) = 4x^2 - 4$ *	A1	www NB **AG**
				[2]	
		OR	$4\tan^2\theta = 4\sec^2\theta - 4$	B1*	Correct substitution of x and y into the given answer
			$\Rightarrow 1 + \tan^2\theta = \sec^2\theta$ which is true	B1dep*	Dependent on previous mark – must simplify/remove the factor of 4 from each term and state that the correctly derived trig identity is true
1	(iii)		$V = \pi\int_1^2 y^2\,dx = \pi\int_1^2 (4x^2 - 4)\,dx$	M1	$k\pi\int_1^2 \left(4x^2 - 4\right)(dx)$ with $k = 1$ or $1/2$, allow correct limits later condone lack of dx
			$\frac{4}{3}x^3 - 4x$	B1	$(4/3)x^3 - 4x$ $\left(\text{or } (2/3)x^3 - 2x\right)$
			$\pi\left[\frac{4}{3}x^3 - 4x\right]_1^2 = \frac{16}{3}\pi$	A1	exact – mark **final** answer
				[3]	

Question			Answer	Marks	Guidance
2			cosec x + 5 cot x = 3 sin x		
			$\Rightarrow \quad \frac{1}{\sin x}+\frac{5\cos x}{\sin x}=3\sin x$	M1	**using** cosec x = 1/sin x and cot x = cos x / sin x
			$\Rightarrow$ 1 + 5 x = 3 ^2x = 1 – cos^2x)	M1	c ^2x + sin^2x = 1 **used** (both M marks must be part of same solution in order to score both marks)
			$\Rightarrow$ 3 cos^2x + 5 cos x – 2 = 0 *	A1	**AG** (**Accept working backwards, with same stages needed**)
			$\Rightarrow$ (3 cos x – 1)(cos x + 2) = 0	M1	use of correct quadratic equation formula (can be an error when substituting into correct formula) or factorising (giving correct coeffs 3 and -2 when multiplied out) or comp square oe
			$\Rightarrow$ cos x = 1/3,	A1	cos x = 1/3 www
			x = 70.5°, 289.5°	A1 A1	for 70.5° or first correct solution, condone over-specification (ie 70.5° or better eg 70.53°,70.5288° etc), for 289.5° or second correct solution (condone over-specification) and no others in the range Ignore solutions outside the range **SC**A1A0 for incorrect answers that round to 70.5 and 360-their ans, eg 70.52 and 289.48 **SC** Award A1A0 for 1.2,5.1 radians (or better) Do not award SC marks if there are extra solutions in the range
				[7]	

Question			Answer	Marks	Guidance
3			 tan 45° = 1/1 = 1*  tan 30° = 1/√3*		For both B marks **AG** so need to be convinced and need triangles but further explanation need not be on their diagram. Any given lengths must be consistent.
				B1	Need √2 or indication that triangle is isosceles oe
				B1	Need all three sides oe
			tan 75° = tan (45°+30°)	M1	use of **correct** compound angle formula with 45º,30º soi
			$= \dfrac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30} = \dfrac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}}$	A1	substitution in terms of √3 in any **correct** form
			$= \dfrac{1+\sqrt{3}}{-1+\sqrt{3}}$	M1	eliminating fractions within a fraction (or rationalising, whichever comes first) provided compound angle formula is used as tan(A+B)= tan(A±B)/(1±tan$A$tan$B$).
			$= \dfrac{(1+\sqrt{3})^2}{3-1}$ (oe eg $\dfrac{3+\sqrt{3}}{3-\sqrt{3}} = \dfrac{(3+\sqrt{3})^2}{9-3}$)	M1	rationalising denominator (or eliminating fractions whichever comes second)
			$= \dfrac{(3+2\sqrt{3}+1)}{3-1} = 2+\sqrt{3}$ *	A1	**correct** only, **AG** so need to see working
				[7]	

Question			Answer	Marks	Guidance
4			LHS = $\sec^2\theta + \operatorname{cosec}^2\theta$		
			$=\dfrac{1}{\cos^2\theta}+\dfrac{1}{\sin^2\theta}$	M1	**Use** of $\sec\theta = 1/\cos\theta$ and $\operatorname{cosec}\theta = 1/\sin\theta$ not just stating
			$=\dfrac{\sin^2\theta+\cos^2\theta}{\cos^2\theta\sin^2\theta}$	M1	adding
			$=\dfrac{1}{\cos^2\theta\sin^2\theta}$	A1	use of $\cos^2\theta + \sin^2\theta = 1$ soi
			$=\sec^2\theta \operatorname{cosec}^2\theta$	A1	**AG**
			…………………………………………………………………		
			OR		
			$\sec^2\theta+\operatorname{cosec}^2\theta=\tan^2\theta+1+\cot^2\theta+1=\sin^2\theta/\cos^2\theta+\cos^2\theta/\sin^2\theta+2$	M1	correct formulae oe
			$=\dfrac{\cos^4\theta+\sin^4\theta+2\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$	M1	adding
			$=\dfrac{(\cos^2\theta+\sin^2\theta)^2}{\sin^2\theta\cos^2\theta}=\dfrac{1}{\sin^2\theta\cos^2\theta}=\sec^2\theta\operatorname{cosec}^2\theta$	A1 A1	use of Pythagoras **AG**
			…………………………………………………………………		
			OR working with both sides		
			Eg LHS $\sec^2\theta+\operatorname{cosec}^2\theta=\tan^2\theta+1+\cot^2\theta+1=\tan^2\theta+\cot^2\theta+2$	M1	Correct formulae used on one side
			RHS $=(1+\tan^2\theta)(1+\cot^2\theta)= 1+\tan^2\theta+\cot^2\theta+\tan^2\theta\cot^2\theta$	M1	Use of same formulae on other side
			$=\tan^2\theta+\cot^2\theta+2=$ LHS	A1	Use of $\tan\theta\cot\theta = 1$ oe, dependent on both method marks
				A1	Showing equal
				[4]	

5 $\text{cosec}^2\theta = 1 + \cot^2\theta$ $\Rightarrow 1 + \cot^2\theta = 1 + 2\cot\theta$ $\Rightarrow \cot^2\theta - 2\cot\theta = 0$ $\Rightarrow \cot\theta(\cot\theta - 2) = 0$ $\Rightarrow \cot\theta = 0$, **and** $\cot\theta = 2$, $\tan\theta = ½$ $\Rightarrow \theta = 26.6°, –153.4°,-90°,90°$ …………………………………………………….. **OR** $\frac{1}{\sin^2\theta} = 1 + \frac{2\cos\theta}{\sin\theta} = \frac{\sin\theta + 2\cos\theta}{\sin\theta}$ $\Rightarrow \sin^2\theta + 2\sin\theta\cos\theta - 1 = 0$ $\Rightarrow 2\sin\theta\cos\theta - \cos^2\theta = 0$ $\Rightarrow \cos\theta\ (2\sin\theta - \cos\theta) = 0$ $\Rightarrow \cos\theta = 0$, **and** $\tan\theta = ½$ $\theta = 26.6°, –153.4°, –90°, 90°$	M1 M1 M1 B3,2,1,0 M1 M1 M1 B3,2,1,0 [6]	correct trig identity used factorising oe **both** needed and $\cot\theta = 1/\tan\theta$ soi –90°, 90°, 27°, –153° or better www …………………………………… correct trig equivalents and a one line equation (or common denominator) formed use of Pythagoras and factorising **both** needed and $\tan\theta = \sin\theta/\cos\theta$ oe soi accept 27º, -153º as above ………………………………………. answers, no working, award B3,2,1,0 (it is possible to score say M1 then B3 ow)	(use of $1-\cot^2\theta$ could lead to M0 M1 M1 B1) allow if $\cot\theta = 0$ not seen (ie quadratic equation followed by $\cot\theta - 2 = 0$ or $\cot\theta = 2$) (omission of $\cot\theta = 0$ could gain M1, M1, M0, B1) …………………………………………... as above allow if $\cos\theta = 0$ not seen (as above) ……………………………………… **in both cases**, -1 if **extra** solutions in the range are given (dependent on at least B1 being scored)-not their incorrect solutions eg 26.6°,-153.4°, 0°,180°,-180° would obtain B1 -1 MR if answers given in radians (-π/2,π/2,0.464, -2.68 (-1.57.1.57) or multiples of π that round to these, or better) (dependent on at least B1 being scored) to lose both of these, at least B2 would need to be scored.

6 $\text{cosec}^2\theta = 1 + \cot^2\theta$ $\Rightarrow$ $1 + \cot^2\theta - \cot\theta = 3$ * $\Rightarrow$ $\cot^2\theta - \cot\theta - 2 = 0$ $\Rightarrow$ $(\cot\theta - 2)(\cot\theta + 1) = 0$ $\Rightarrow$ $\cot\theta = 2$, $\tan\theta = ½$, $\theta = 26.57°$ $\cot\theta = -1$, $\tan\theta = -1$, $\theta = 135°$	E1 M1 A1 M1 A1 A1 [6]	clear use of $1+\cot^2\theta = \text{cosec}^2\theta$ factorising or formula roots 2, –1 cot = 1/tan used $\theta = 26.57°$ $\theta = 135°$ (penalise extra solutions in the range (-1))

7 $\sec\theta = x/2$, $\tan\theta = y/3$ $\sec^2\theta = 1 + \tan^2\theta$ $\Rightarrow$ $x^2/4 = 1 + y^2/9$ $\Rightarrow$ $x^2/4 - y^2/9 = 1$ * **OR** $x^2/4 - y^2/9 = 4\sec^2\theta/4 - 9\tan^2\theta/9$ $= \sec^2\theta - \tan^2\theta = 1$	M1 M1 E1 [3]	$\sec^2\theta = 1 + \tan^2\theta$ used (oe, e.g. converting to sines and cosines and using $\cos^2\theta + \sin^2\theta = 1$) eliminating θ (or x and y) www

8 $\sec^2\theta = 4$ $\Rightarrow$ $\dfrac{1}{\cos^2\theta} = 4$ $\Rightarrow$ $\cos^2\theta = ¼$ $\Rightarrow$ $\cos\theta = ½$ or $-½$ $\Rightarrow$ $\theta = \pi/3, 2\pi/3$ **OR** $\sec^2\theta = 1 + \tan^2\theta$ $\Rightarrow$ $\tan^2\theta = 3$ $\Rightarrow$ $\tan\theta = \sqrt{3}$ or $-\sqrt{3}$ $\Rightarrow$ $\theta = \pi/3, 2\pi/3$	**M1** **M1** **A1 A1** **M1** **M1** **A1 A1** **[4]**	sec θ = 1/cos θ used ± ½ allow unsupported answers ± √3 allow unsupported answers