1 $\operatorname{cosec} = 1 + \cot^2 \theta$ $\Rightarrow$ $1 + \cot^2 \theta - \cot \theta = 3$ * $\Rightarrow$ $\cot^2 \theta - \cot \theta - 2 = 0$ $\Rightarrow$ $(\cot \theta - 2)(\cot \theta + 1) = 0$ $\Rightarrow$ $\cot \theta = 2$, $\tan \theta = ½$, $\theta = 26.57°$ $\cot \theta = -1$, $\tan \theta = -1$, $\theta = 135°$	E1 M1 A1 M1 A1 A1 [6]	clear use of $1+\cot^2\theta = \operatorname{cosec}^2\theta$ factorising or formula roots 2, –1 cot = 1/tan used $\theta = 26.57°$ $\theta = 135°$ (penalise extra solutions in the range (-1))

2(i) (*A*) 360°÷24=15° CB/OB = sin 15° ⇒ CB = 1 sin 15° ⇒ AB = 2CB = 2 sin 15°*	 M1 E1 [2]	AB=2AC or 2CB ∠AOC= 15° oe
(*B*) cos 30° = 1 – 2 $\sin^2$ 15° cos 30° = √3/2 ⇒ √3/2 = 1 – 2 $\sin^2$ 15° ⇒ 2 $\sin^2$ 15° = 1 – √3/2 = (2 – √3)/2 ⇒ $\sin^2$15° = (2 – √3)/4 ⇒ sin 15° = $\sqrt{\frac{2-\sqrt{3}}{4}} = \frac{1}{2}\sqrt{2-\sqrt{3}}$ *	B1 B1 M1 E1 [4]	 simplifying
(*C*) Perimeter = 12 × AB = 24 × ½ √(2 – √3) = 12√(2 – √3) circumference of circle > perimeter of polygon ⇒ 2π > 12√(2 – √3) ⇒ π > 6√(2 – √3)	M1 E1 [2]	
(ii) (*A*) tan 15° = FE/OF ⇒ FE = tan 15° ⇒ DE = 2FE = 2tan 15°	M1 E1 [2]	
(*B*) $\tan 30 = \frac{2\tan 15}{1-\tan^2 15} = \frac{2t}{1-t^2}$ tan 30 = 1/√3 ⇒ $\frac{2t}{1-t^2} = \frac{1}{\sqrt{3}} \Rightarrow 2\sqrt{3}t = 1-t^2$ ⇒ $t^2 + 2\sqrt{3}\,t - 1 = 0$ *	B1 M1 E1 [3]	
(*C*) $t = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2} = 2-\sqrt{3}$ circumference < perimeter ⇒ 2π < 24(2 – √3) ⇒ π < 12(2 – √3) *	M1 A1 M1 E1 [4]	 using positive root from exact working
(iii) 6√(2 – √3) < π < 12(2 – √3) ⇒ 3.106 < π < 3.215	 B1 B1 [2]	 3.106, 3.215

3 sin θ − 3 cos θ = R sin(θ − α) = R(sin θ cos α − cos θ sin α) ⇒ R cos α = 1 , R sin α = 3 ⇒ $R^2 = 1^2 + 3^2 = 10 \Rightarrow R = \sqrt{10}$ tan α = 3 ⇒ α = 71.57° $\sqrt{10}\sin(\theta - 71.57°) = 1$ ⇒ $\theta - 71.57° = \sin^{-1}(1/\sqrt{10})$ θ − 71.57°= 18.43°, 161.57° ⇒ θ = 90° , 233.1°	M1 B1 M1 A1 M1 B1 A1 [7]	equating correct pairs oe ft www cao (71.6°or better) oe ft R, α www and no others in range (MR-1 for radians)

4 (i) $\frac{dy}{dx}=\frac{10\cos\theta+10\cos 2\theta}{-10\sin\theta-10\sin 2\theta}$ $=-\frac{\cos\theta+\cos 2\theta}{\sin\theta+\sin 2\theta}$ * When $\theta=\pi/3$, $\frac{dy}{dx}=-\frac{\cos\pi/3+\cos 2\pi/3}{\sin\pi/3+\sin 2\pi/3}$ $= 0$ as $\cos\pi/3 = ½$, $\cos 2\pi/3 = -½$ At A $x = 10\cos\pi/3 + 5\cos 2\pi/3$ $= 2½$ $y = 10\sin\pi/3 + 5\sin 2\pi/3 = 15\sqrt{3}/2$	M1 E1 B1 M1 A1 A1 [6]	$dy/d\theta \div dx/d\theta$ or solving $\cos\theta+\cos 2\theta=0$ substituting $\pi/3$ into x or y 2½ $15\sqrt{3}/2$ (condone 13 or better)
(ii) $x^2+y^2=(10\cos\theta+5\cos 2\theta)^2+(10\sin\theta+5\sin 2\theta)^2$ $=100\cos^2\theta+100\cos\theta\cos 2\theta+25\cos^2 2\theta$ $+100\sin^2\theta+100\sin\theta\sin 2\theta+25\sin^2 2\theta$ $= 100 + 100\cos(2\theta-\theta) + 25$ $= 125 + 100\cos\theta$ *	B1 M1 DM1 E1 [4]	 expanding $\cos 2\theta\cos\theta+\sin 2\theta\sin\theta=\cos(2\theta-\theta)$ or substituting for $\sin 2\theta$ and $\cos 2\theta$
(iii) Max $\sqrt{125+100} = 15$ min $\sqrt{125-100} = 5$	B1 B1 [2]	
(iv) $2\cos^2\theta+2\cos\theta-1=0$ $\cos\theta=\frac{-2\pm\sqrt{12}}{4}=\frac{-2\pm 2\sqrt{3}}{4}$ At B, $\cos\theta=\frac{-1+\sqrt{3}}{2}$ $OB^2 = 125 + 50(-1+\sqrt{3}) = 75 + 50\sqrt{3} = 161.6…$ $\Rightarrow$ $OB = \sqrt{161.6…} = 12.7$ (m)	 M1 A1 M1 A1 [4]	 quadratic formula or θ=68.53° or 1.20radians, correct root selected or OB=$10\sin\theta+5\sin 2\theta$ ft their $\theta/\cos\theta$ oe cao

5		$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ $\Rightarrow \quad \cot 2\theta = \frac{1}{\tan 2\theta} = \frac{1-\tan^2\theta}{2\tan\theta}*$ $\cot 2\theta = 1 + \tan\theta$ $\Rightarrow \quad \frac{1-\tan^2\theta}{2\tan\theta} = 1+\tan\theta$ $\Rightarrow \quad 1-\tan^2\theta = 2\tan\theta + 2\tan^2\theta$ $\Rightarrow \quad 3\tan^2\theta + 2\tan\theta - 1 = 0$ $\Rightarrow \quad (3\tan\theta - 1)(\tan\theta + 1) = 0$ $\Rightarrow \quad \tan\theta = 1/3$, $\theta = 18.43°, 198.43°$ or $\tan\theta = -1$, $\theta = 135°, 315°$	M1 E1 M1 M1 A3,2,1,0 **[7]**	oe eg converting either side into a one line fraction(s) involving sin θ and cos θ. quadratic = 0 factorising or solving 18.43°, 198.43°, 135°, 315° –1 extra solutions in the range