1	**(i)**	BAC = 120 – 90 – (90 – θ) = θ – 60 $\Rightarrow$ BC = b sin(θ – 60) CD = AE = a sin θ $\Rightarrow$ h = BC + CD = a sin θ + b sin (θ – 60°) *	B1 M1 E1 **[3]**	
	(ii)	h = a sin θ + b sin (θ – 60°) = a sin θ + b (sinθ cos 60 – cosθ sin 60) = a sin θ + ½ b sin θ – √3/2 b cos θ $= (a+\frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta$ *	M1 M1 E1 **[3]**	corr compound angle formula sin 60 = √3/2, cos 60 = ½ used
	(iii)	OB horizontal when h = 0 $\Rightarrow$ $(a+\frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta = 0$ $\Rightarrow$ $(a+\frac{1}{2}b)\sin\theta = \frac{\sqrt{3}}{2}b\cos\theta$ $\Rightarrow$ $\frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{3}}{2}b}{a+\frac{1}{2}b}$ $\Rightarrow$ $\tan\theta = \frac{\sqrt{3}b}{2a+b}$ *	M1 M1 E1 **[3]**	$\frac{\sin\theta}{\cos\theta} = \tan\theta$
	(iv)	$2\sin\theta - \sqrt{3}\cos\theta = R\sin(\theta - \alpha)$ $= R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$ $\Rightarrow$ $R\cos\alpha = 2$, $R\sin\alpha = \sqrt{3}$ $\Rightarrow$ $R^2 = 2^2 + (\sqrt{3})^2 = 7$, $R = \sqrt{7} = 2.646$ m tan α = √3/2, α = 40.9° So $h = \sqrt{7}\sin(\theta - 40.9°)$ $\Rightarrow$ $h_{max} = \sqrt{7} = 2.646$ m when θ – 40.9° = 90° $\Rightarrow$ θ = 130.9°	M1 B1 M1A1 B1ft M1 A1 **[7]**	

2 $3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$ $= R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 3, R\sin\alpha = 4$ $\Rightarrow R^2 = 3^2 + 4^2 = 25 \Rightarrow R = 5$ $\tan\alpha = 4/3 \Rightarrow \alpha = 0.9273$ $5\cos(\theta - 0.9273) = 2$ $\Rightarrow \cos(\theta - 0.9273) = 2/5$ $\theta - 0.9273 = 1.1593, -1.1593$ $\Rightarrow \theta = 2.087, -0.232$	M1 B1 M1A1 M1 A1 A1 [7]	 $R = 5$ cwo and no others in the range

Question			Answer	Marks	Guidance
3			$\text{cosec } x + 5 \cot x = 3 \sin x$		
			$\Rightarrow \quad \frac{1}{\sin x} + \frac{5\cos x}{\sin x} = 3\sin x$	M1	**using** $\text{cosec } x = 1/\sin x$ and $\cot x = \cos x / \sin x$
			$\Rightarrow \quad$ **1** + 5 x = 3 ^2x = $1 - \cos^2 x)$	**M1**	c $^2x + \sin^2 x = 1$ **used** (both M marks must be part of same solution in order to score both marks)
			$\Rightarrow \quad 3\cos^2 x + 5\cos x - 2 = 0$ *	A1	**AG** (**Accept working backwards, with same stages needed**)
			$\Rightarrow \quad (3\cos x - 1)(\cos x + 2) = 0$	M1	use of correct quadratic equation formula (can be an error when substituting into correct formula) or factorising (giving correct coeffs 3 and -2 when multiplied out) or comp square oe
			$\Rightarrow \quad \cos x = 1/3,$	A1	$\cos x = 1/3$ www
			$x = 70.5°,$	A1	for 70.5° or first correct solution, condone over-specification (ie
			289.5°	A1	70.5° or better eg 70.53°,70.5288° etc),
					for 289.5° or second correct solution (condone over-specification) and no others in the range
					Ignore solutions outside the range
					SCA1A0 for incorrect answers that round to 70.5 and 360-their ans, eg 70.52 and 289.48
					SC Award A1A0 for 1.2,5.1 radians (or better)
					Do not award SC marks if there are extra solutions in the range
				[7]	

4(i) At A, $y = 0$ x-coord of A $= 2\times\pi/2 - \sin\pi/2 = \pi - 1$ $\Rightarrow$ x-coord of B $= 2\times\pi - \sin\pi = 2\pi$ AC $= 2\pi - \pi + 1 = \pi + 1$ $\Rightarrow$ θ ratio is $(\pi - 1):(\pi + 1)$ * At B, $\theta = \pi$	B1 B1 M1 A1 E1 [5]	 for either A or B/C for both A and B/C
(ii) $\dfrac{dy}{d\theta} = -4\sin\theta$ $\dfrac{dx}{d\theta} = 2 - \cos\theta$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$ $= -\dfrac{4\sin\theta}{2-\cos\theta}$ At A, gradient $= -\dfrac{4\sin(\pi/2)}{2-\cos(\pi/2)} = -2$	B1 M1 A1 A1 [4]	either $dx/d\theta$ or $dy/d\theta$ www
(iii) $\dfrac{dy}{dx} = 1 \Rightarrow -\dfrac{4\sin\theta}{2-\cos\theta} = 1$ $\Rightarrow$ $-4\sin\theta = 2 - \cos\theta$ $\Rightarrow$ $\cos\theta - 4\sin\theta = 2$ *	M1 E1 [2]	their $dy/dx = 1$
(iv) $\cos\theta - 4\sin\theta = R\cos(\theta + \alpha)$ $= R(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$ $\Rightarrow$ $R\cos\alpha = 1$, $R\sin\alpha = 4$ $\Rightarrow$ $R^2 = 1^2 + 4^2 = 17$, $R = \sqrt{17}$ $\tan\alpha = 4$, $\alpha = 1.326$ $\Rightarrow$ $\sqrt{17}\cos(\theta + 1.326) = 2$ $\Rightarrow$ $\cos(\theta + 1.326) = 2/\sqrt{17}$ $\Rightarrow$ $\theta + 1.326 = 1.064, 5.219, 7.348$ $\Rightarrow$ $\theta = (-0.262), 3.89, 6.02$	M1 B1 M1 A1 M1 A1 A1 [7]	corr pairs accept 76.0^o , 1.33 radians inv cos $(2/\sqrt{17})$ ft their R for method -1 extra solutions in the range

5	$4\cos\theta - \sin\theta = R\cos(\theta + \alpha)$ $= \quad \theta\cos\alpha - R\sin\quad \sin\alpha$ $\Rightarrow R\cos\alpha = 4, \quad R\sin\alpha = 1$ $\Rightarrow R^2 = 1^2 + 4^2 = 17, R = \sqrt{17} = 4.123$ $\tan\alpha = ¼$ $\Rightarrow \alpha = 0.245$ $\sqrt{17}\cos(\theta + 0.245) = 3$ $\Rightarrow \cos(\theta + 0.245) = 3/\sqrt{17}$ $\Rightarrow \theta + 0.245 = 0.756, 5.527$ $\Rightarrow \theta = 0.511, 5.282$	M1 $R\cos$ B1 M1 A1 M1 A1A1 [7]	correct pairs θ $R = \sqrt{17} = 4.123$ $\tan\alpha = ¼$ o.e. $\alpha = 0.245$ $\theta + 0.245 = \arccos 3/\sqrt{17}$ ft their R, α for method (penalise extra solutions in the range (-1))