1 Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence find the range of the function $f(\theta)$, where

$$f(\theta) = 7 + 3\cos\theta + 4\sin\theta \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

Write down the greatest possible value of $\dfrac{1}{7 + 3\cos\theta + 4\sin\theta}$. [6]

2 Express $3\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence find, correct to 2 decimal places, the coordinates of the maximum point on the curve $y = f(x)$, where

$$f(x) = 3\sin x + 2\cos x, \; 0 \leqslant x \leqslant \pi.$$ **[7]**

3 Show that $\dfrac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$. **[3]**

4 The angle θ satisfies the equation $\sin(\theta + 45°) = \cos\theta$.

(i) Using the exact values of $\sin 45°$ and $\cos 45°$, show that $\tan\theta = \sqrt{2} - 1$. [5]

(ii) Find the values of θ for $0° < \theta < 360°$. [2]

5 Solve the equation $2\sin 2\theta + \cos 2\theta = 1$, for $0° \leqslant \theta < 360°$. [6]

6 Express $6\cos 2\theta + \sin\theta$ in terms of $\sin\theta$.

Hence solve the equation $6\cos 2\theta + \sin\theta = 0$, for $0° \leqslant \theta \leqslant 360°$. **[7]**

7 **(i)** Show that $\cos(\alpha + \beta) = \dfrac{1 - \tan\alpha\tan\beta}{\sec\alpha\sec\beta}$. **[3]**

(ii) Hence show that $\cos 2\alpha = \dfrac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$. **[2]**

(iii) Hence or otherwise solve the equation $\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta} = \dfrac{1}{2}$ for $0° \leqslant \theta \leqslant 180°$. **[3]**

8 In Fig. 6, OAB is a thin bent rod, with OA = a metres, AB = b metres and angle OAB = 120°. The bent rod lies in a vertical plane. OA makes an angle θ above the horizontal. The vertical height BD of B above O is h metres. The horizontal through A meets BD at C and the vertical through A meets OD at E.

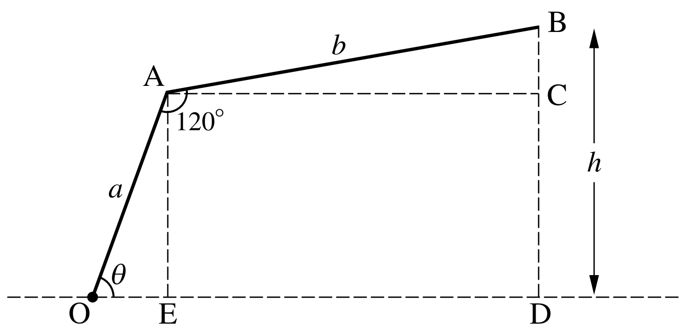

Fig. 6

(i) Find angle BAC in terms of θ. Hence show that

$$h = a\sin\theta + b\sin(\theta - 60°).$$ **[3]**

(ii) Hence show that $h = (a + \frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta$. **[3]**

The rod now rotates about O, so that θ varies. You may assume that the formulae for h in parts **(i)** and **(ii)** remain valid.

(iii) Show that OB is horizontal when $\tan\theta = \dfrac{\sqrt{3}b}{2a+b}$. **[3]**

In the case when $a = 1$ and $b = 2$, $h = 2\sin\theta - \sqrt{3}\cos\theta$.

(iv) Express $2\sin\theta - \sqrt{3}\cos\theta$ in the form $R\sin(\theta - \alpha)$. Hence, for this case, write down the maximum value of h and the corresponding value of θ. **[7]**

9 **(i)** Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is acute, expressing α in terms of π. **[4]**

(ii) Write down the derivative of $\tan\theta$.

Hence show that $\displaystyle\int_0^{\frac{1}{3}\pi} \frac{1}{(\cos\theta + \sqrt{3}\sin\theta)^2}\,d\theta = \frac{\sqrt{3}}{4}$. **[4]**