1	$3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$ $= R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = 4$ $\Rightarrow R^2 = 3^2 + 4^2 = 25,\ R = 5$ $\tan\alpha = 4/3 \Rightarrow$ $\alpha = 0.927$ $f(\theta) = 7 + 5\cos(\theta - 0.927)$ $\Rightarrow$ Range is 2 to 12 Greatest value of $\frac{1}{7+3\cos\theta+4\sin\theta}$ is ½ .	**B1** **M1** **A1** **M1** **A1** **B1ft** **[6]**	R=5 tan α=4/3 oe ft their R 0.93 or 53.1° or better their cos (θ - 0. 927) = 1 or -1 used (*condone use of graphical calculator)* 2 and 12 seen cao simplified

2			$3\sin x + 2\cos x = R\sin(x + \alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$ $\Rightarrow$ $R\cos\alpha = 3,\ R\sin\alpha = 2$ $\Rightarrow$ $R^2 = 3^2 + 2^2 = 13,\ R = \sqrt{13}$ $\tan\alpha = 2/3$, $\alpha = 0.588$ $\Rightarrow$ $3\sin x + 2\cos x = \sqrt{13}\sin(x + 0.588)$ maximum when $x + 0.588 = \pi/2$ $\Rightarrow$ $x = \pi/2 - 0.588 = 0.98$ rads $\Rightarrow$ $y = \sqrt{13} = 3.61$ So coords of max point are (0.98, 3.61)	M1 B1 M1 A1 M1 A1 B1 **[7]**	Correct pairs. Condone omission of R if used correctly. Condone sign error. or 3.6 or better, not ±√13 unless+√13 chosen ft from first M1 0.588 or better (accept 0.59), with no errors seen in method for angle (allow 33.7º or better) any valid method eg differentiating 0.98 only. Do not accept degrees or multiples of π. condone √13, ft their R if ,say =√14

3 LHS = $\dfrac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$ $= \dfrac{2\sin\theta\cos\theta}{2\cos^2\theta}$ $= \dfrac{\sin\theta}{\cos\theta} = \tan\theta = \text{RHS}$	M1 M1 E1 [3]	one correct double angle formula used cancelling cos θ's

4(i) sin(θ + 45°) = cos θ ⇒ sin θ cos 45 + cos θ sin 45 = cos θ ⇒ (1/√2) sin θ + (1/√2) cos θ = cos θ ⇒ sin θ + cos θ = √2 cos θ ⇒ sin θ = (√2 – 1) cos θ ⇒ $\dfrac{\sin\theta}{\cos\theta}$ = tan θ = √2 – 1 *	M1 B1 A1 M1 E1 [5]	compound angle formula sin 45 = 1/√2, cos 45 = 1/√2 collecting terms
(ii) tan θ = √2 – 1 ⇒ θ = 22.5°, 202.5°	B1 B1 [2]	 and no others in the range

5(i) 2sin 2θ + cos 2θ = 1 ⇒ 4sin θ cos θ + 1 – 2sin² θ = 1 ⇒ 2sin θ(2cos θ – sin θ) = 0 or 4 tanθ-2tan²θ=0 ⇒ sin θ = 0 or tan θ =0, θ = 0°, 180° or 2c θ – sin θ = 0 ⇒ tan θ = 2 ⇒ θ = 63.43°, 243.43° OR Using Rsin(2θ+α) R=√5 and α=26.57° 2θ +26.57=arcsin 1/R θ=0° ,180° θ=63.43°, 243.43°	M1 A1 A1 M1 A1, A1 [6] M1 A1 M1 A1 A1,A1 [6]	Using double angle formulae Correct simplification to factorisable or other form that leads to solutions 0° and 180° tan θ = 2 (-1 for extra solutions in range) (-1 for extra solutions in range)

Question			Answer	Marks	Guidance
6			$\cos 2\theta = 1 - 2\sin^2\theta$	M1*	$\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ (maybe implied in substitution)
			$(6\cos 2\theta + \sin\theta =)\ 6 - 12\sin^2\theta + \sin\theta$	A1	
			$6\cos 2\theta + \sin\theta = 0$ $\Rightarrow \quad 12\sin^2\theta - \sin\theta - 6 = 0$ $\Rightarrow \quad (4\sin\theta - 3)(3\sin\theta + 2) = 0$	M1dep*	Use of correct quadratic equation formula or factorising or comp. the square on their three term quadratic in $\sin\theta$ (see guidance in question 1 for awarding this method mark) provided $b^2 - 4ac \geq 0$
			$\Rightarrow \quad \sin\theta = 3/4$ or $-2/3$	A1	www
				B1	First correct solution to 1 dp or better (eg 48.59° etc)
				B1	Three correct solutions
			$\Rightarrow \quad \sin\theta = 3/4,\ \theta = 48.6°, 131.4°$ $\sin\theta = -2/3,\ \theta = 221.8°, 318.2°$	B1	All four correct solutions and no others in the range
					Ignore solutions outside the range
					SC Award max B1B1B0 for answers in radians (0.85, 2.29, 3.87, 5.55 or better – so one correct B1, three correct B1). Award max B1 if there are extra solutions in the range with radians
					SC If M1M1 awarded and both values of $\lvert\sin\theta\rvert \leq 1$ but B0B0B0 then award B1 only for evidence of using $\sin\theta \equiv \sin(180 - \theta)$
				[7]	

Question			Answer	Marks	Guidance
7	(i)		EITHER Use of cos = 1/sec (or sin= 1/cosec)	B1	Must be **used**
			From RHS $\frac{1-\tan\alpha\tan\beta}{\sec\alpha\sec\beta}$ $=\frac{1-\sin\alpha/\cos\alpha.\sin\beta/\cos\beta}{1/\cos\alpha.1/\cos\beta}$ $=\cos\alpha\cos\beta(1-\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta})$	M1	Substituting and simplifying as far as having no fractions within a fraction [need more than $\frac{1-tt}{\sec\sec}=cc-ss$ ie an intermediate step that can lead to cc–ss]
			$=\cos\alpha\cos\beta-\sin\alpha\sin\beta$ $=\cos(\alpha+\beta)$	A1	Convincing simplification and correct use of $\cos(\alpha+\beta)$ Answer given
			OR From LHS, cos = 1/sec or sin = 1/cosec **used**	B1	
			$\cos(\alpha+\beta)$ $=\cos\alpha\cos\beta-\sin\alpha\sin\beta$ $=\frac{1}{\sec\alpha\sec\beta}-\sin\alpha\sin\beta$ $=\frac{1-\sec\alpha\sin\alpha\sec\beta\sin\beta}{\sec\alpha\sec\beta}$	M1	Correct angle formula and substitution and simplification to one term OR eg $\cos\alpha\cos\beta-\sin\alpha\sin\beta$ $=\cos\alpha\cos\beta(1-\tan\alpha\tan\beta)$
			$=\frac{1-\tan\alpha\tan\beta}{\sec\alpha\sec\beta}$	A1	Simplifying to final answer www **Answer given**
				[3]	Or any equivalent work but must have more than cc–ss = answer.

Question			Answer	Marks	Guidance
7	(ii)		$\beta = \alpha$ $\cos 2\alpha = \frac{1-\tan^2\alpha}{\sec^2\alpha}$ $= \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$.	M1 A1	$\beta = \alpha$ used , Need to see $\sec^2\alpha$ Use of $\sec^2\alpha = 1 + \tan^2\alpha$ to give required result Answer Given
			OR, without Hence, $\cos 2\alpha = \cos^2\alpha(1 - \frac{\sin^2\alpha}{\cos^2\alpha})$ $= \frac{1}{\sec^2\alpha}(1-\tan^2\alpha)$ $= \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$	M1 **[2]**	Use of $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ soi Simplifying and using $\sec^2\alpha = 1+ \tan^2\alpha$ to final answer Answer Given Accept working in reverse to show RHS=LHS, or showing equivalent
7	(iii)		$\cos 2\theta = ½$ $2\theta = 60°, 300°$ $\theta = 30°, 150°$	M1 A1 A1 **[3]**	Soi or from $\tan^2\theta = 1/3$ oe from $\sin^2\theta$ or $\cos^2\theta$ First correct solution Second correct solution and no others in the range SC B1 for $\pi/6$and $5\pi/6$ and no others in the range

8	**(i)**	BAC = 120 – 90 – (90 – θ) = θ – 60 $\Rightarrow$ BC = b sin(θ – 60) CD = AE = a sin θ $\Rightarrow$ h = BC + CD = a sin θ + b sin (θ – 60°) *	B1 M1 E1 **[3]**	
	(ii)	$h = a \sin\theta + b\sin(\theta - 60°)$ $= a\sin\theta + b(\sin\theta\cos 60 - \cos\theta\sin 60)$ $= a\sin\theta + ½ b\sin\theta - \sqrt{3}/2\, b\cos\theta$ $= (a + \frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta$ *	M1 M1 E1 **[3]**	corr compound angle formula sin 60 = √3/2, cos 60 = ½ used
	(iii)	OB horizontal when h = 0 $\Rightarrow (a + \frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta = 0$ $\Rightarrow (a + \frac{1}{2}b)\sin\theta = \frac{\sqrt{3}}{2}b\cos\theta$ $\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{3}}{2}b}{a + \frac{1}{2}b}$ $\Rightarrow \tan\theta = \frac{\sqrt{3}b}{2a + b}$ *	M1 M1 E1 **[3]**	$\frac{\sin\theta}{\cos\theta} = \tan\theta$
	(iv)	$2\sin\theta - \sqrt{3}\cos\theta = R\sin(\theta - \alpha)$ $= R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 2,\ R\sin\alpha = \sqrt{3}$ $\Rightarrow R^2 = 2^2 + (\sqrt{3})^2 = 7,\ R = \sqrt{7} = 2.646$ m $\tan\alpha = \sqrt{3}/2,\ \alpha = 40.9°$ So $h = \sqrt{7}\sin(\theta - 40.9°)$ $\Rightarrow h_{max} = \sqrt{7} = 2.646$ m when $\theta - 40.9° = 90°$ $\Rightarrow \theta = 130.9°$	 M1 B1 M1A1 B1ft M1 A1 **[7]**	

9(i) $\cos\theta + \sqrt{3}\sin\theta = r\cos(\theta - \alpha)$ $= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = \sqrt{3}$ $\Rightarrow R^2 = 1^2 + (\sqrt{3})^2 = 4,\ R = 2$ $\tan\alpha = \sqrt{3}$ $\Rightarrow \alpha = \pi/3$	B1 M1 M1 A1 [4]	$R = 2$ equating correct pairs $\tan\alpha = \sqrt{3}$ o.e.
(ii) derivative of tan θ is $\sec^2\theta$ $\int_0^{\frac{\pi}{3}} \frac{1}{(\cos\theta + \sqrt{3}\sin\theta)^2} d\theta = \int_0^{\frac{\pi}{3}} \frac{1}{4}\sec^2(\theta - \frac{\pi}{3}) d\theta$ $= \left[\frac{1}{4}\tan(\theta - \frac{\pi}{3})\right]_0^{\frac{\pi}{3}}$ $= \frac{1}{4}(0 - (-\sqrt{3}))$ $= \sqrt{3}/4$ *	B1 M1 A1 E1 [4]	 ft their α $\frac{1}{R^2}$ [tan ($\theta - \pi/3$] ft their R,α(in radians) www