1 Express $2\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants to be determined, and $0 < \alpha < \frac{1}{2}\pi$.

Hence write down the greatest and least possible values of $1 + 2\sin\theta - 3\cos\theta$. **[6]**

2 Express $4\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $4\cos\theta - \sin\theta = 3$, for $0 \leqslant \theta \leqslant 2\pi$. **[7]**

3 Archimedes, about 2200 years ago, used regular polygons inside and outside circles to obtain approximations for π.

(i) Fig. 8.1 shows a regular 12-sided polygon inscribed in a circle of radius 1 unit, centre O. AB is one of the sides of the polygon. C is the midpoint of AB. Archimedes used the fact that the circumference of the circle is greater than the perimeter of this polygon.

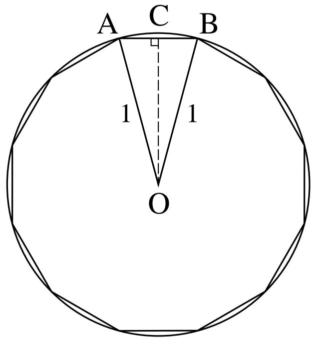

Fig. 8.1

(A) Show that AB = $2\sin 15°$. **[2]**

(B) Use a double angle formula to express $\cos 30°$ in terms of $\sin 15°$. Using the exact value of $\cos 30°$, show that $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$. **[4]**

(C) Use this result to find an exact expression for the perimeter of the polygon.

Hence show that $\pi > 6\sqrt{2-\sqrt{3}}$. **[2]**

(ii) In Fig. 8.2, a regular 12-sided polygon lies outside the circle of radius 1 unit, which touches each side of the polygon. F is the midpoint of DE. Archimedes used the fact that the circumference of the circle is less than the perimeter of this polygon.

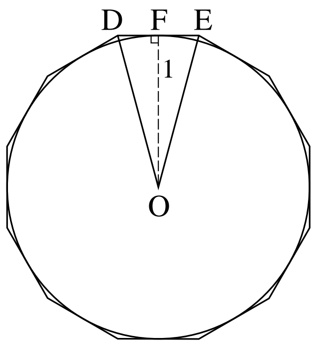

Fig. 8.2

(A) Show that DE = $2\tan 15°$. **[2]**

(B) Let $t = \tan 15°$. Use a double angle formula to express $\tan 30°$ in terms of t.

Hence show that $t^2 + 2\sqrt{3}t - 1 = 0$. **[3]**

(C) Solve this equation, and hence show that $\pi < 12\left(2-\sqrt{3}\right)$. **[4]**

(iii) Use the results in parts **(i)**(C) and **(ii)**(C) to establish upper and lower bounds for the value of π, giving your answers in decimal form. **[2]**

4 Solve the equation $\cos 2\theta = \sin\theta$ for $0 \leqslant \theta \leqslant 2\pi$, giving your answers in terms of π. [7]

5 Express $\sqrt{3}\sin x - \cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Express α in the form $k\pi$.

Find the exact coordinates of the maximum point of the curve $y = \sqrt{3}\sin x - \cos x$ for which $0 < x < 2\pi$. [6]

6 Express $\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants to be determined, and $0° < \alpha < 90°$.

Hence solve the equation $\sin\theta - 3\cos\theta = 1$ for $0° \leqslant \theta \leqslant 360°$. [7]

7 Fig. 1 shows part of the graph of $y = \sin x \quad \sqrt{3}\cos x$.

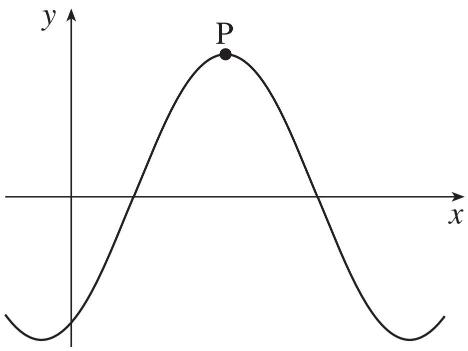

Fig. 1

Express $\sqrt{\ }$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 \leqslant \alpha \leqslant \frac{1}{2}\pi$.

Hence write down the exact coordinates of the turning point P. [6]