1	$2\sin\theta - 3\cos\theta = R\sin(\theta - \alpha)$ $= R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 2,\ R\sin\alpha = 3$ $\Rightarrow R^2 = 2^2 + 3^2 = 13,\ R = \sqrt{13}$ $\tan\alpha = 3/2,$ $\Rightarrow \alpha = 0.983$ minimum $1 - \sqrt{13}$, maximum $1 + \sqrt{13}$	M1 B1 M1 A1 B1 B1 [6]	correct pairs $R = \sqrt{13}$ or 3.61 or better 0.98 or better or −2.61, 4.61 or better	condone wrong sign at this stage correct division, ft from first M1 radians only accept multiples of π that round to 0.98 allow B1, B1ft for 1-√R and 1+√R for their R to 2dp or better

2 $4\cos\theta - \sin\theta = R\cos(\theta + \alpha)$ $= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 4,\ R\sin\alpha = 1$ $\Rightarrow R^2 = 1^2 + 4^2 = 17$, $R = \sqrt{17} = 4.123$ tan α = ¼ $\Rightarrow \alpha = 0.245$ $\sqrt{17}\cos(\theta + 0.245) = 3$ $\Rightarrow \cos(\theta + 0.245) = 3/\sqrt{17}$ $\Rightarrow \theta + 0.245 = 0.756, 5.527$ $\Rightarrow \theta = 0.511, 5.282$	M1 B1 M1 A1 M1 A1A1 [7]	correct pairs $R = \sqrt{17} = 4.123$ tan α = ¼ o.e. $\alpha = 0.245$ $\theta + 0.245 = \arccos 3/\sqrt{17}$ ft their R, α for method (penalise extra solutions in the range (-1))

3(i) (*A*) $360°\div24=15°$ CB/OB = sin 15° $\Rightarrow$ CB = 1 sin 15° $\Rightarrow$ AB = 2CB = 2 sin 15°*	M1 E1 [2]	AB=2AC or 2CB ∠AOC= 15° oe
(*B*) $\cos 30° = 1 - 2\sin^2 15°$ $\cos 30° = \sqrt{3}/2$ $\Rightarrow \sqrt{3}/2 = 1 - 2\sin^2 15°$ $\Rightarrow 2\sin^2 15° = 1 - \sqrt{3}/2 = (2 - \sqrt{3})/2$ $\Rightarrow \sin^2 15° = (2 - \sqrt{3})/4$ $\Rightarrow \sin 15° = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{1}{2}\sqrt{2-\sqrt{3}}$ *	B1 B1 M1 E1 [4]	simplifying
(*C*) Perimeter = $12 \times AB = 24 \times \frac{1}{2}\sqrt{(2-\sqrt{3})}$ $= 12\sqrt{(2-\sqrt{3})}$ circumference of circle > perimeter of polygon $\Rightarrow 2\pi > 12\sqrt{(2-\sqrt{3})}$ $\Rightarrow \pi > 6\sqrt{(2-\sqrt{3})}$	M1 E1 [2]	
(ii) (*A*) tan 15° = FE/OF $\Rightarrow$ FE = tan 15° $\Rightarrow$ DE = 2FE = 2tan 15°	M1 E1 [2]	
(*B*) $\tan 30 = \frac{2\tan 15}{1-\tan^2 15} = \frac{2t}{1-t^2}$ $\tan 30 = 1/\sqrt{3}$ $\Rightarrow \frac{2t}{1-t^2} = \frac{1}{\sqrt{3}} \Rightarrow 2\sqrt{3}t = 1 - t^2$ $\Rightarrow t^2 + 2\sqrt{3}\,t - 1 = 0$ *	B1 M1 E1 [3]	
(*C*) $t = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2} = 2 - \sqrt{3}$ circumference < perimeter $\Rightarrow 2\pi < 24(2-\sqrt{3})$ $\Rightarrow \pi < 12(2-\sqrt{3})$ *	M1 A1 M1 E1 [4]	using positive root from exact working
(iii) $6\sqrt{(2-\sqrt{3})} < \pi < 12(2-\sqrt{3})$ $\Rightarrow 3.106 < \pi < 3.215$	B1 B1 [2]	3.106, 3.215

4 $\cos 2\theta = \sin\theta$ $\Rightarrow 1 - 2\sin^2\theta = \sin\theta$ $\Rightarrow 1 - \sin\theta - 2\sin^2\theta = 0$ $\Rightarrow (1 - 2\sin\theta)(1 + \sin\theta) = 0$ $\Rightarrow \sin\theta = ½$ or -1 $\Rightarrow \theta = \pi/6, 5\pi/6, 3\pi/2$	M1 M1 A1 M1 A1 A2,1,0 [7]	$\cos 2\theta = 1 - 2\sin^2\theta$ oe substituted forming quadratic(in one variable) =0 correct quadratic www factorising or solving quadratic ½ , −1 oe www cao penalise extra solutions in the range

5 $\sqrt{3}\sin x - \cos x = R\sin(x - \alpha)$ $= R(\sin x\cos\alpha - \cos x\sin\alpha)$ $\Rightarrow \sqrt{3} = R\cos\alpha, 1 = R\sin\alpha$ $\Rightarrow R^2 = 3 + 1 = 4 \Rightarrow R = 2$ $\tan\alpha = 1/\sqrt{3}$ $\Rightarrow \alpha = \pi/6$ $\Rightarrow y = 2\sin(x - \pi/6)$ Max when $x - \pi/6 = \pi/2 \Rightarrow x = \pi/6 + \pi/2 = 2\pi/3$ max value $y = 2$ So maximum is $(2\pi/3, 2)$	M1 B1 M1 A1 B1 B1 [6]	correct pairs soi $R = 2$ ft cao www cao ft their R SC B1 $(2, 2\pi/3)$ no working

6 $\sin\theta - 3\cos\theta = R\sin(\theta - \alpha)$ $= R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 1, R\sin\alpha = 3$ $\Rightarrow R^2 = 1^2 + 3^2 = 10 \Rightarrow R = \sqrt{10}$ $\tan\alpha = 3 \Rightarrow \alpha = 71.57°$ $\sqrt{10}\sin(\theta - 71.57°) = 1$ $\Rightarrow \theta - 71.57° = \sin^{-1}(1/\sqrt{10})$ $\theta - 71.57° = 18.43°, 161.57°$ $\Rightarrow \theta = 90°,$ $233.1°$	M1 B1 M1 A1 M1 B1 A1 [7]	equating correct pairs oe ft www cao (71.6°or better) oe ft R, α www and no others in range (MR-1 for radians)

7 $\sin x - \sqrt{3}\cos x = R\sin(x - \alpha)$ $= R(\sin x \cos \alpha - \cos x \sin \alpha)$ $\Rightarrow$ $R\cos\alpha = 1$, $R\sin\alpha = \sqrt{3}$ $\Rightarrow$ $R^2 = 1^2 + (\sqrt{3})^2 = 4$, $R = 2$ $\tan\alpha = \sqrt{3}/1 = \sqrt{3} \Rightarrow \alpha = \pi/3$ $\Rightarrow \sin x - \sqrt{3}\cos x = 2\sin(x - \pi/3)$ x coordinate of P is when $x - \pi/3 = \pi/2$ $\Rightarrow x = 5\pi/6$ $y = 2$ So coordinates are $(5\pi/6, 2)$	B1 M1 A1 M1 A1ft B1ft [6]	$R = 2$ $\tan\alpha = \sqrt{3}$ or $\sin\alpha = \sqrt{3}$/their R or $\cos\alpha = 1$/their R $\alpha = \pi/3$, 60° or 1.05 (or better) radians www Using x-their $\alpha = \pi/2$or 90° $\alpha \neq 0$ exact radians only (not $\pi/2$) their R (exact only)