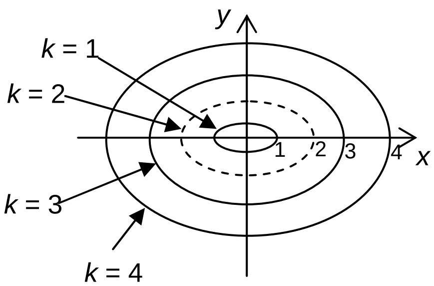
1(i) $\frac{dy}{d\theta} = 2\cos 2\theta - 2\sin\theta$, $\frac{dx}{d\theta} = 2\cos\theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\cos 2\theta - 2\sin\theta}{2\cos\theta} = \frac{\cos 2\theta - \sin\theta}{\cos\theta}$	B1, B1 M1 A1 [4]	substituting for theirs oe
(ii) When $\theta = \pi/6$, $\frac{dy}{dx} = \frac{\cos\pi/3 - \sin\pi/6}{\cos\pi/6}$ $= \frac{1/2 - 1/2}{\sqrt{3}/2} = 0$ Coords of B: $x = 2 + 2\sin(\pi/6) = 3$ $y = 2\cos(\pi/6) + \sin(\pi/3) = 3\sqrt{3}/2$ $BC = 2\times 3\sqrt{3}/2 = 3\sqrt{3}$	E1 M1 A1,A1 B1ft [5]	for either exact
(iii) (*A*) $y = 2\cos\theta + \sin 2\theta$ $= 2\cos\theta + 2\sin\theta\cos\theta$ $= 2\cos\theta(1 + \sin\theta)$ $= x\cos\theta$ * (*B*) si $\theta = ½(x - 2)$ $\cos^2\theta = 1 - \sin^2\theta$ $= 1 - ¼(x-2)^2$ $= 1 - ¼x^2 + x - 1$ $= (x - ¼x^2)$ * (*C*) artesian equation is $y^2 = x^2\cos^2\theta$ $= x^2(x - ¼x^2)$ $= x^3 - ¼x^4$*	M1 E1 B1 M1 E1 M1 E1 [7]	$\sin 2\theta = 2\sin\theta\cos\theta$ squaring and substituting for *x*
(iv) $V = \int_0^4 \pi y^2 dx$ $= \pi\int_0^4 (x^3 - \frac{1}{4}x^4)dx$ $= \pi\left[\frac{1}{4}x^4 - \frac{1}{20}x^5\right]_0^4$ $= \pi(64 - 51.2)$ $= 12.8\pi = 40.2$ (m^3)	M1 B1 A1 [3]	need limits $\left[\frac{1}{4}x^4 - \frac{1}{20}x^5\right]$ 12.8π or 40 or better.

2(i) $\cos\theta = \frac{x}{k}, \sin\theta = \frac{2y}{k}$ $\cos^2\theta + \sin^2\theta = 1$ $\Rightarrow \left(\frac{x}{k}\right)^2 + \left(\frac{2y}{k}\right)^2 = 1$ $\Rightarrow \frac{x^2}{k^2} + \frac{4y^2}{k^2} = 1$ $\Rightarrow x^2 + 4y^2 = k^2$ *	M1 M1 E1 [3]	Used substitution
(ii) $\frac{dx}{d\theta} = -k\sin\theta, \frac{dy}{d\theta} = \frac{1}{2}k\cos\theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{\frac{1}{2}k\cos\theta}{k\sin\theta}$ $= -½ \cot\theta$ $-\frac{x}{4y} = -\frac{2k\cos\theta}{4k\sin\theta} = -\frac{1}{2}\cot\theta = \frac{dy}{dx}$	M1 A1 E1	 oe
or, by differentiating implicitly $2x + 8y\, dy/dx = 0$ $\Rightarrow dy/dx = -2x/8y = -x/4y$*	M1 A1 E1 [3]	
(iii) $k = 2$	B1 [1]	
(iv) 	B1 B1 B1 [3]	1 correct curve –shape and position 2 or more curves correct shape- in concentric form all 3 curves correct
(v) grad of stream path = –1/grad of contour $\Rightarrow \frac{dy}{dx} = -\frac{1}{(-x/4y)} = \frac{4y}{x}$ *	M1 E1 [2]	
(vi) $\frac{dy}{dx} = \frac{4y}{x} \Rightarrow \int\frac{dy}{y} = \int\frac{4dx}{x}$ $\Rightarrow \ln y = 4\ln x + c = \ln e^c x^4$ $\Rightarrow y = Ax^4$ where $A = e^c$. When $x = 2, y = 1 \Rightarrow 1 = 16A \Rightarrow A = 1/16$ $\Rightarrow y = x^4/16$ *	M1 A1 M1 M1 A1 E1 [6]	Separating variables $\ln y = 4\ln x\ (+c)$ antilogging correctly (at any stage) substituting $x = 2, y = 1$ evaluating a correct constant www

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3	$dx/dt = 1 - 1/t$ $dy/dt = 1 + 1/t$ $\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $= \frac{1+\frac{1}{t}}{1-\frac{1}{t}}$ When $t = 2$, $dy/dx = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3$	B1 M1 A1 M1 A1 [5]	Either dx/dt or dy/dt soi www

Question			Answer	Marks	Guidance
4	(i)		$\theta = -\pi/2$: O (0, 0) $\theta = 0$: P (2, 0) $\theta = \pi/2$: O (0, 0)	B1 B1 B1 **[3]**	Origin or O, condone omission of (0, 0) or O Or, say at P $x = 2$, $y = 0$, need P stated Origin or O, condone omission of (0,0) or O
4	(ii)		$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{2\cos 2\theta}{-2\sin\theta} = -\frac{\cos 2\theta}{\sin\theta}$ When $\theta = \pi/2$ $dy/dx = -\cos \pi/ \sin\pi/2 = 1$ When $\theta = -\pi/2$ $dy/dx = -\cos(-\pi)/ \sin(-\pi/2) = -1$ **Either** $1\times -1 = -1$ so perpendicular **Or** gradient tangent $=1 \Rightarrow$ meets axis at 45º , similarly, gradient $= -1 \Rightarrow$ meets axis at 45º oe	M1 A1 M1 A1 A1 **[5]**	their $dy/d\theta$ /$dx/d\theta$ any equivalent form www (not from $-2\cos 2\theta/2\sin\theta$) subst $\theta = \pi/2$ in their equation Obtaining $dy/dx = 1$,and $dy/dx = -1$ **shown** (or explaining using symmetry of curve) **www** justification that tangents are perpendicular **www** dependent on previous A1
4	(iii)		At Q, $\sin 2\theta = 1 \Rightarrow 2\theta = \pi/2$, $\theta = \pi/4$ $\Rightarrow$ coordinates of Q are $(2\cos \pi/4, \sin \pi/2)$ $= (\sqrt{2}, 1)$	M1 A1 A1 **[3]**	or, using the derivative, $\cos 2\theta = 0$ soi or their $dy/dx = 0$ to find θ. If the only error is in the sign or the coeff of the derivative in (ii), allow full marks in this part (condone $\theta = 45°$) www (exact only) accept $2/\sqrt{2}$
4	(iv)		$\sin^2\theta = (1 - \cos^2\theta) = 1 - ¼ x^2$ $\Rightarrow$ $y = \sin 2\theta = 2\sin\theta\cos\theta$ $= (\pm)\, x\sqrt{(1 - ¼ x^2)}$ $\Rightarrow$ $y^2 = x^2(1 - ¼ x^2)$*	B1 M1 A1 A1 **[4]**	oe, eg may be $x^2 =$….. **Use** of $\sin 2\theta = 2\sin\theta\cos\theta$ subst for x **or** $y^2 = 4\sin^2\theta\cos^2\theta$ (squaring) either order oe squaring **or** subst for x either order oe **AG**

Question			Answer	Marks	Guidance
4	(v)		$V = \int_0^2 \pi x^2 (1 - \frac{1}{4}x^2)\,\mathrm{d}x$	M1	integral including correct limits but ft their '2' from (i) (limits may appear later) condone omission of d*x* if intention clear
			$= \int_0^2 (\pi x^2 - \frac{1}{4}\pi x^4)\,\mathrm{d}x$		
			$= \pi\left[\frac{1}{3}x^3 - \frac{1}{20}x^5\right]_0^2$	B1	$\left[\frac{1}{3}x^3 - \frac{1}{20}x^5\right]$ ie allow if no π and/or incorrect/no limits (or equivalent by parts)
			$= \pi\left[\frac{8}{3} - \frac{32}{20}\right]$	A1	substituting limits into correct expression (including π) ft their '2'
			$= 16\pi/15$	A1 **[4]**	cao oe, 3.35 or better (any multiple of π must round to 3.35 or better)
4	(vi)		$\overrightarrow{AA'} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$	B1	finding $\overrightarrow{AA'}$ *or* $\overrightarrow{A'A}$ by subtraction, **subtraction must be seen** B0 if $\overrightarrow{AA'}$, $\overrightarrow{A'A}$ confused Assume they have found $\overrightarrow{AA'}$ if no label
			This vector is normal to $x + 2y - 3z = 0$	B1	reference to normal or ***n***, or perpendicular to $x + 2y - 3z = 0$, or statement that vector matches coefficients of plane and is therefore perpendicular, or showing AA' is perpendicular to **two** vectors in the plane
			M is (1½, 3, 2½) $x + 2y - 3z = 1½ + 6 - 7½ = 0$	M1	for finding M correctly (can be implied by two correct coordinates)
			$\Rightarrow$ M lies in plane	A1 **[4]**	showing numerical subst of M in plane = 0

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5(i) $\frac{3}{(y-2)(y+1)} = \frac{A}{y-2} + \frac{B}{y+1}$ $= \frac{A(y+1)+B(y-2)}{(y-2)(y+1)}$ $\Rightarrow$ $3 = A(y + 1) + B(y - 2)$ $y = 2 \Rightarrow 3 = 3A \Rightarrow A = 1$ $y = -1 \Rightarrow 3 = -3B \Rightarrow B = -1$	 M1 A1 A1 [3]	 substituting, equating coeffs or cover up
(ii) $\frac{dy}{dx} = x^2(y-2)(y+1)$ $\Rightarrow \int \frac{3\,\mathrm{d}y}{(y-2)(y+1)} = \int 3x^2\,\mathrm{d}x$ $\Rightarrow \int (\frac{1}{(y-2)} - \frac{1}{y+1})\,\mathrm{d}y = \int 3x^2\,\mathrm{d}x$ $\Rightarrow \ln(y - 2) - \ln(y + 1) = x^3 + c$ $\Rightarrow \ln\left(\frac{y-2}{y+1}\right) = x^3 + c$ $\Rightarrow \frac{y-2}{y+1} = e^{x^3+c} = e^{x^3}.e^c = Ae^{x^3}$ *	 M1 B1ft B1 M1 E1 [5]	 separating variables $\ln(y - 2) - \ln(y + 1)$ ft their A,B $x^3 + c$ anti-logging including c www