1 (i) $\frac{dy}{dx}=\frac{10\cos\theta+10\cos 2\theta}{-10\sin\theta-10\sin 2\theta}$ $= -\frac{\cos\theta+\cos 2\theta}{\sin\theta+\sin 2\theta}$ * When $\theta = \pi/3$, $\frac{dy}{dx} = -\frac{\cos\pi/3+\cos 2\pi/3}{\sin\pi/3+\sin 2\pi/3}$ $= 0$ as $\cos \pi/3 = ½$, $\cos 2\pi/3 = -½$ At A $x = 10\cos\pi/3 + 5\cos 2\pi/3$ $= 2½$ $y = 10\sin\pi/3 + 5\sin 2\pi/3 = 15\sqrt{3}/2$	M1 E1 B1 M1 A1 A1 [6]	$dy/d\theta \div dx/d\theta$ or solving $\cos\theta+\cos 2\theta=0$ substituting $\pi/3$ into x or y 2½ $15\sqrt{3}/2$ (condone 13 or better)
(ii) $x^2+y^2=(10\cos\theta+5\cos 2\theta)^2+(10\sin\theta+5\sin 2\theta)^2$ $=100\cos^2\theta+100\cos\theta\cos 2\theta+25\cos^2 2\theta$ $+100\sin^2\theta+100\sin\theta\sin 2\theta+25\sin^2 2\theta$ $= 100 + 100\cos(2\theta - \theta) + 25$ $= 125 + 100\cos\theta$ *	B1 M1 DM1 E1 [4]	 expanding $\cos 2\theta\cos\theta + \sin 2\theta\sin\theta = \cos(2\theta - \theta)$ or substituting for $\sin 2\theta$ and $\cos 2\theta$
(iii) Max $\sqrt{125+100} = 15$ min $\sqrt{125-100} = 5$	B1 B1 [2]	
(iv) $2\cos^2\theta + 2\cos\theta - 1 = 0$ $\cos\theta = \frac{-2\pm\sqrt{12}}{4} = \frac{-2\pm 2\sqrt{3}}{4}$ At B, $\cos\theta = \frac{-1+\sqrt{3}}{2}$ $OB^2 = 125 + 50(-1+\sqrt{3}) = 75 + 50\sqrt{3} = 161.6\ldots$ $\Rightarrow$ $OB = \sqrt{161.6\ldots} = 12.7$ (m)	 M1 A1 M1 A1 [4]	 quadratic formula or θ=68.53° or 1.20radians, correct root selected or OB=$10\sin\theta+5\sin 2\theta$ ft their $\theta/\cos\theta$ oe cao

2 (i) At E, $\theta = 2\pi$ $\Rightarrow$ $x = a(2\pi - \sin 2\pi) = 2a\pi$ So OE = $2a\pi$. Max height is when $\theta = \pi$ $\Rightarrow$ $y = a(1 - \cos\pi) = 2a$	M1 A1 M1 A1 [4]	$\theta=\pi$, 180°,$\cos\theta=-1$
(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{a\sin\theta}{a(1-\cos\theta)}$ $= \frac{\sin\theta}{(1-\cos\theta)}$	M1 M1 A1 [3]	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ for theirs $\frac{d}{d\theta}(\sin\theta) = \cos\theta$, $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$ both or equivalent www condone uncancelled a
(iii) $\tan 30° = 1/\sqrt{3}$ $\Rightarrow$ $\frac{\sin\theta}{(1-\cos\theta)} = \frac{1}{\sqrt{3}}$ $\Rightarrow$ $\sin\theta = \frac{1}{\sqrt{3}}(1-\cos\theta)$ * When $\theta = 2\pi/3$, $\sin\theta = \sqrt{3}/2$ $(1 - \cos\theta)/\sqrt{3} = (1 + ½)/\sqrt{3} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$ BF = $a(1 + ½) = 3a/2$* OF = $a(2\pi/3 - \sqrt{3}/2)$	M1 E1 M1 E1 E1 B1 [6]	Or gradient=$1/\sqrt{3}$ $\sin\theta = \sqrt{3}/2$, $\cos\theta = -½$ or equiv.
(iv) BC = $2a\pi - 2a(2\pi/3 - \sqrt{3}/2)$ $= a(2\pi/3 + \sqrt{3})$ AF = $\sqrt{3} \times 3a/2 = 3\sqrt{3}a/2$ AD = BC + 2AF $= a(2\pi/3 + \sqrt{3} + 3\sqrt{3})$ $= a(2\pi/3 + 4\sqrt{3})$ $= 20$ $\Rightarrow a = 2.22$ m	B1ft M1 A1 M1 A1 [5]	their OE -2their OF

3 (i) $y^2 - x^2 = (t + 1/t)^2 - (t - 1/t)^2$ $= t^2 + 2 + 1/t^2 - t^2 + 2 - 1/t^2$ $= 4$	**M1** **E1** **[2]**	Substituting for x and y in terms of t oe
(ii) EITHER $dx/dt = 1 + 1/t^2$, $dy/dt = 1 - 1/t^2$ $\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $= \frac{1-1/t^2}{1+1/t^2}$ $= \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$ * **OR** $2y\frac{dy}{dx} - 2x = 0$ $\Rightarrow \frac{dy}{dx} = \frac{x}{y} = \frac{t-1/t}{t+1/t}$ $= \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$ **OR** $y=\sqrt{(4+x^2)}$, $\Rightarrow \frac{dy}{dx} = \frac{x}{\sqrt{4+x^2}}$ $= \frac{t-1/t}{\sqrt{4+t^2-2+1/t^2}}$ $= \frac{t-1/t}{\sqrt{(t+1/t^2)}} = \frac{t-1/t}{(t+1/t)}$ $= \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$ $\Rightarrow$ $dy/dx = 0$ when $t = 1$ or -1 $t = 1, \Rightarrow (0, 2)$ $t = -1 \Rightarrow (0, -2)$	**B1** **M1** **E1** **B1** **M1** **E1** **B1** **M1** **E1** **M1** **A1 A1** **[6]**	For both results

Question			Answer	Marks	Guidance
4	(i)		$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{2\cos 2\theta}{\cos\theta}$	M1 A1	their dy/dθ / their dx/dθ www correct (can isw)
			When θ = π/6 = $\frac{dy}{dx}=\frac{2\cos(\pi/3)}{\cos(\pi/6)}$	DM1	subst $\theta = \pi/6$ in theirs
			$=\frac{1}{\sqrt{3}/2}=\frac{2}{\sqrt{3}}$	A1	oe exact only, www (but not 1/√3/2)
			…………………………………………………………		………………………………………………………………………………………..
			OR		
			$y=2x\sqrt{(1-x^2)}$ $\frac{dy}{dx}=-2x^2(1-x^2)^{-1/2}+2(1-x^2)^{1/2}$	M1 A1	full method for differentiation including product rule and function of a function oe oe cao (condone lack of consideration of sign)
			$at\theta=\pi/6, \sin\pi/6=1/2$	DM1	subst sin π/6 = ½ in theirs
			$\frac{dy}{dx}=\frac{-2}{4}(1-\frac{1}{4})^{-1/2}+2(\frac{3}{4})^{1/2}=\frac{2}{\sqrt{3}}$	A1	oe ,exact only, www (but not 1/√3/2)
				[4]	
4	(ii)		$y = \sin 2\theta = 2\sin\theta\cos\theta$	M1	using $\sin 2\theta = 2\sin\theta\cos\theta$
			$\Rightarrow \quad y^2 = 4\sin^2\theta\cos^2\theta = 4x^2(1-x^2)$ $= 4x^2 - 4x^4$ *	M1 A1 **[3]**	using $\cos^2\theta = 1 - \sin^2\theta$ to eliminate $\cos\theta$ **AG** need to see sufficient working or A0.

5	$x = \frac{1}{1+t} \Rightarrow 1+t = \frac{1}{x}$	M1	attempt to solve for *t*
$\Rightarrow$	$t = \frac{1}{x} - 1$	A1	oe
	$y = \frac{1-t}{1+2t} = \frac{1-\frac{1}{x}+1}{1+\frac{2}{x}-2}$	M1	substituting for *t* in terms of *x*
	$= \frac{2-\frac{1}{x}}{\frac{2}{x}-1} = \frac{2x-1}{2-x}$	M1 A1 [5]	clearing subsidiary fractions

6	**(i)**	$\frac{dy}{dt} = \frac{(1+t).2 - 2t.1}{(1+t)^2} = \frac{2}{(1+t)^2}$	M1A1	
		$\frac{dx}{dt} = 2e^{2t}$	B1	
		$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$	M1	
		$\Rightarrow \frac{dy}{dx} = \frac{\frac{2}{(1+t)^2}}{2e^{2t}} = \frac{1}{e^{2t}(1+t)^2}$	A1	
		$t = 0 \Rightarrow dy/dx = 1$	B1ft **[6]**	

	(ii)	$2t = \ln x \Rightarrow t = ½ \ln x$	M1	or *t* in terms of *y*
		$\Rightarrow y = \frac{\ln x}{1+\frac{1}{2}\ln x} = \frac{2\ln x}{2+\ln x}$	A1 **[2]**	