1 You are given that $f(x) = \cos x + \lambda \sin x$ where λ is a positive constant.

(i) Express $f(x)$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving R and α in terms of λ. **[4]**

(ii) Given that the maximum value (as x varies) of $f(x)$ is 2, find R, λ and α, giving your answers in exact form. **[4]**

2 Fig. 7 shows the curve BC defined by the parametric equations

$$x = 5\ln u,\ y = u + \frac{1}{u}, \quad 1 \leqslant u \leqslant 10.$$

The point A lies on the x-axis and AC is parallel to the y-axis. The tangent to the curve at C makes an angle θ with AC, as shown.

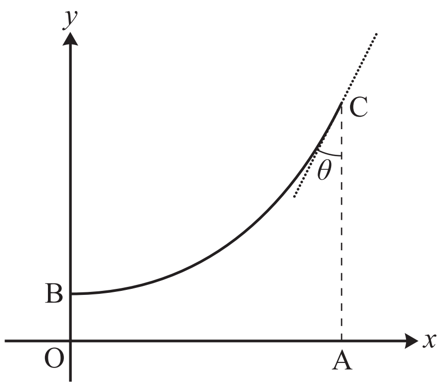

Fig. 7

(i) Find the lengths OA, OB and AC. **[5]**

(ii) Find $\frac{dy}{dx}$ in terms of u. Hence find the angle θ. **[6]**

(iii) Show that the cartesian equation of the curve is $y = e^{\frac{1}{5}x} + e^{-\frac{1}{5}x}$. **[2]**

An object is formed by rotating the region OACB through 360° about Ox.

(iv) Find the volume of the object. **[5]**

3 A curve has parametric equations

$$x = 2\sin\theta, \quad y = \cos 2\theta.$$

(i) Find the exact coordinates and the gradient of the curve at the point with parameter $\theta = \frac{1}{3}\pi$. **[5]**

(ii) Find y in terms of x. **[2]**

4 The parametric equations of a curve are

$$x = \cos 2\theta, \quad y = \sin\theta\cos\theta \quad \text{for } 0 \leqslant \theta < \pi.$$

Show that the cartesian equation of the curve is $x^2 + 4y^2 = 1$.

Sketch the curve. **[5]**

5 Part of the track of a roller-coaster is modelled by a curve with the parametric equations

$$x = 2\theta - \sin\theta, \quad y = 4\cos\theta \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

This is shown in Fig. 8. B is a minimum point, and BC is vertical.

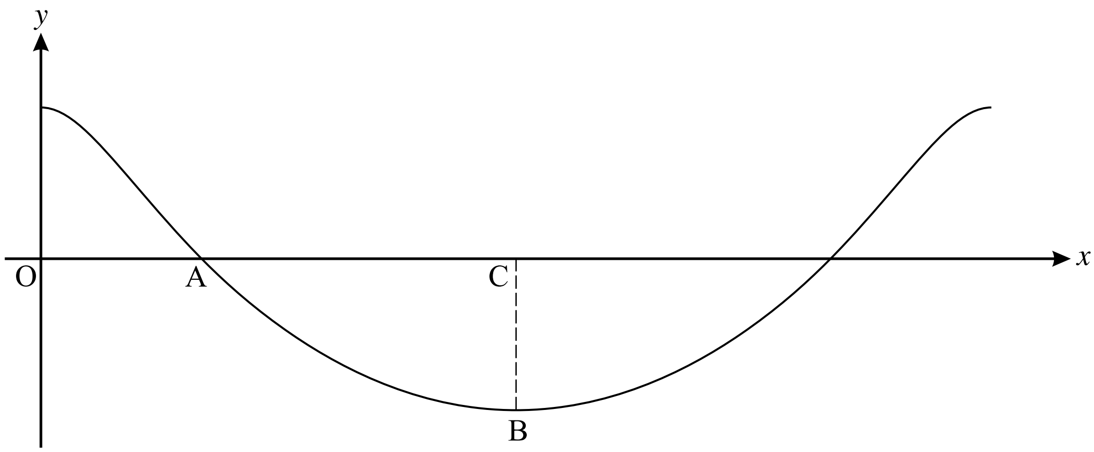

Fig. 8

(i) Find the values of the parameter at A and B.

Hence show that the ratio of the lengths OA and AC is $(\pi - 1) : (\pi + 1)$. **[5]**

(ii) Find $\frac{dy}{dx}$ in terms of θ. Find the gradient of the track at A. **[4]**

(iii) Show that, when the gradient of the track is 1, θ satisfies the equation

$$\cos\theta - 4\sin\theta = 2. \quad \textbf{[2]}$$

(iv) Express $\cos\theta - 4\sin\theta$ in the form $R\cos(\theta + \alpha)$.

Hence solve the equation $\cos\theta - 4\sin\theta = 2$ for $0 \leqslant \theta \leqslant 2\pi$. **[7]**

6 A curve has parametric equations

$$x = at^3, \quad y = \frac{a}{1+t^2},$$

where a is a constant.

Show that $\frac{dy}{dx} = \frac{-2}{3t(1+t^2)^2}$.

Hence find the gradient of the curve at the point $(a, \frac{1}{2}a)$. **[7]**

7 A curve has parametric equations $x = 1 + u^2$, $y = 2u^3$.

(i) Find $\frac{dy}{dx}$ in terms of u. [3]

(ii) Hence find the gradient of the curve at the point with coordinates (5, 16). [2]

8 A curve is defined by parametric equations

$$x = \frac{1}{t} - 1, \; y = \frac{2+t}{1+t}.$$

Show that the cartesian equation of the curve is $y = \frac{3+2x}{2+x}$. [4]