Question			Answer	Marks	Guidance
1	(i)		$\cos x + \lambda \sin x = R\cos(x-\alpha)$ $= R\cos x\cos\alpha + R\sin x\sin\alpha$		
			$\Rightarrow \quad R\cos\alpha = 1,\ R\sin\alpha = \lambda$	M1	Correct pairs. Condone sign error (so accept $R\sin\alpha = -\lambda$)
			$\Rightarrow \quad R^2 = 1+\lambda^2,\ R = \sqrt{(1+\lambda^2)}$	B1	Positive square root only - isw. Accept $R = 1/\cos(\arctan\lambda)$ or $R = \lambda/\sin(\arctan\lambda)$
			$\tan\alpha = \lambda$ (oe)	M1	Follow through their pairs. $\tan\alpha = \lambda$ with no working implies both M marks. However, $\cos\alpha = 1$, $\sin\alpha = \lambda \Rightarrow \tan\alpha = \lambda$ scores M0M1. First two M marks may be implied by combining one of the pairs with R, eg, $\cos\alpha = \frac{1}{\sqrt{(1+\lambda^2)}}$ or $\sin\alpha = \frac{\lambda}{\sqrt{(1+\lambda^2)}}$
			$\Rightarrow \alpha = \arctan\lambda$ (oe)	A1	$\alpha = \arccos\left(\frac{1}{\sqrt{(1+\lambda^2)}}\right)$, $\alpha = \arcsin\left(\frac{\lambda}{\sqrt{(1+\lambda^2)}}\right)$ Accept embedded answers, eg, $\sqrt{(1+\lambda^2)}\cos(x-\arctan\lambda)$ for full marks
				[4]	
1	(ii)		max is R so $R = 2$	B1	
			$1+\lambda^2 = 4 \Rightarrow \lambda = \sqrt{3}$	M1 A1	M1 for using their $\sqrt{(1+\lambda^2)} = R_{max}$, A0 for $\pm\sqrt{3}$ as final answer
			$\alpha = \arctan\sqrt{3} = \pi/3$	B1	www (eg $\lambda = 1$ and $\cos\alpha = (1+\lambda)^{-1} \Rightarrow \alpha = \pi/3$ is B0) Exact answers only for final A and B marks
				[4]	

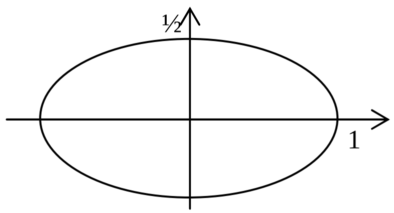
PhysicsAndMathsTutor.com

2	**(i)**		$u = 10$, $x = 5 \ln 10 = 11.5$	M1	Using $u = 10$ to find OA
			so OA = $5\ln 10$	A1	accept 11.5 or better
			when $u = 1$,	M1	Using $u = 1$ to find OB or $u = 10$ to find AC
			$y = 1 + 1 = 2$ so OB = 2	A1	
			When $u = 10$, $y = 10 + 1/10 = 10.1$		
			So AC = 10.1	A1	
					In the case where values are given in coordinates instead of OA=,OB=,AC=, then give A0 on the first occasion this happens but allow subsequent As.
					Where coordinates are followed by length eg B(0, 2), length=2 then allow A1.
				[5]	

Question			Answer	Marks	Guidance
2	(ii)		$\frac{dy}{dx}=\frac{dy/du}{dx/du}=\frac{1-1/u^2}{5/u}$	M1	their dy/du /dx/du
			$\left[=\frac{u^2-1}{5u}\right]$	A1	Award A1 if **any** correct form is seen at any stage including unsimplified (can isw)
			EITHER		
			When u = 10, dy/dx = 99/50 = 1.98	M1	**substituting u =10 in their expression**
			tan (90 – θ)= 1.98 ⇒ θ = 90 – 63.2	M1	**or by geometry**, say using a triangle and the gradient of the line
			= 26.8°	A2	26.8º, or 0.468 radians (or better) cao **SC** M1M0A1A0 for 63.2º (or better) or 1.103 radians(or better)
			OR		
			When u = 10, dy/dx = 99/50 = 1.98	M1	
			tan(90– θ) = 99/50 ⇒ tanθ = 50/99	M1	**allow use of their expression for M marks**
			θ = 26.8º	A2	26.8º, or 0.468 radians (or better) cao
				[6]	
2	(iii)		$x = 5 \ln u \Rightarrow x/5 = \ln u$, $u = e^{x/5}$	M1	Need some working
			$\Rightarrow \quad y = u + 1/u = e^{x/5} + e^{-x/5}$	A1	Need some working as AG
				[2]	

Question			Answer	Marks	Guidance
2	(iv)		Vol of rev = $\int_0^{5\ln 10} \pi y^2 \mathrm{d}x = \int_0^{5\ln 10} \pi(e^{x/5} + e^{-x/5})^2 \mathrm{d}x$	M1	need π $(e^{x/5} + e^{-x/5})^2$ and dx soi. Condone wrong limits or omission of limits for M1. Allow M1 if y prematurely squared as eg $(e^{2x/5} + e^{-2x/5})$
			$= \int_0^{5\ln 10} \pi(\mathrm{e}^{2x/5} + 2 + \mathrm{e}^{-2x/5})\mathrm{d}x$	A1	including **correct** limits at some stage (condone 11.5 for this mark)
			$= \pi\left[(\frac{5}{2}\mathrm{e}^{2x/5} + 2x - \frac{5}{2}\mathrm{e}^{-2x/5})\right]_0^{5\ln 10}$	B1	$[\frac{5}{2}e^{2x/5} + 2x - \frac{5}{2}e^{-2x/5}]$ allow if no π and/or no limits or incorrect limits
			= π(250 + 10ln10 – 0.025 – 0)	M1	substituting both limits (their OA and 0) in an expression of correct form ie $ae^{2x/5} + be^{-2x/5} + cx$, $a,b,c \neq 0$ and subtracting in correct order (- 0 is sufficient for lower limit) Condone absence of π for M1
			= 858	A1	accept 273π and answers rounding to 273π or 858
				[5]	NB The integral can be evaluated using a change of variable to u. This involves changing dx to (dx/du)x du. For completely correct work from this method award full marks. Partially correct solutions must include the change in dx. If in doubt consult your TL. **Remember to indicate second box has been seen even if it has not been used.**

3(i) $x = 2\sin\theta$, $y = \cos 2\theta$ When $\theta = \pi/3$, $x = 2\sin\pi/3 = \sqrt{3}$ $y = \cos 2\pi/3 = -½$ **EITHER** $\mathrm{d}x/\mathrm{d}\theta = 2\cos\theta$, $\mathrm{d}y/\mathrm{d}\theta = -2\sin 2\theta$ $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-\sin 2\theta}{\cos\theta}$ $= \dfrac{-\sin 2\pi/3}{\cos\pi/3} = \dfrac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$ …………………………………………………………... **OR** expressing y in terms of x, $y=1-x^2/2$ $\dfrac{dy}{dx} = -x$ or $-2\sin\theta$ $= -\sqrt{3}$	B1 B1 M1 A1 A1 …….. M1 A1 A1 [5]	$x = \sqrt{3}$ $y = -½$ $\mathrm{d}y/\mathrm{d}x = (\mathrm{d}y/\mathrm{d}\theta) / (\mathrm{d}x/\mathrm{d}\theta)$ used any correct equivalent form exact www ………………………………………… exact www	exact only (isw all dec answers following exact ans) ft their derivatives if right way up (condone one further minor slip if intention clear) condone poor notation can isw if incorrect simplification
(ii) $y = 1 - 2\sin^2\theta = 1 - 2(x/2)^2 = 1 - ½x^2$	M1A1 [2]	or reference to (i) if used there	for M1, need correct trig identity and attempt to substitute for x allow SC B1 for $y=\cos 2\arcsin(x/2)$ or equivalent

4 $y = \sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$	M1	use of sin 2θ
$x = \cos 2\theta$	M1	
$\sin^2 2\theta + \cos^2 2\theta = 1$	E1	
$\Rightarrow x^2 + (2y)^2 = 1$ $\Rightarrow x^2 + 4y^2 = 1$ *		
or $x^2 + 4y^2 = (\cos 2\theta)^2 + 4(\sin\theta\cos\theta)^2$	M1	substitution
$= \cos^2 2\theta + \sin^2 2\theta$	M1	use of $\sin 2\theta$
$= 1$ *	E1	
or $\cos 2\theta = 2\cos^2\theta - 1$ $\cos^2\theta = (x+1)/2$ $\cos 2\theta = 1 - 2\sin^2\theta$ $\sin^2\theta = (1-x)/2$	M1	for both
$y^2 = \sin^2\theta\cos^2\theta = \left(\frac{1-x}{2}\right)\left(\frac{x+1}{2}\right)$	M1	
$y^2 = (1-x^2)/4$ $x^2 + 4y^2 = 1$*	E1	
or $x = \cos 2\theta = \cos^2\theta - \sin^2\theta$ $x^2 = \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta$ $y^2 = \sin^2\theta\cos^2\theta$	M1	correct use of double angle formulae
$x^2 + 4y^2 = \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta + 4\sin^2\theta\cos^2\theta$ $= (\cos^2\theta + \sin^2\theta)^2$	M1	correct squaring and use of $\sin^2\theta + \cos^2\theta = 1$
$= 1$*	E1	
	M1 A1	ellipse correct intercepts
	[5]	

5 (i) At A, $y = 0$ ⇒ $4\cos\theta = 0$, ⇒ $\theta = \pi/2$ At B, $\cos\theta = -1$, ⇒ $\theta = \pi$ x-coord of A $= 2\times\pi/2 - \sin\pi/2 = \pi - 1$ ⇒ x-coord of B $= 2\times\pi - \sin\pi = 2\pi$ OA $= \pi - 1$, AC $= 2\pi - \pi + 1 = \pi + 1$ ratio is $(\pi - 1):(\pi + 1)$ *	B1 B1 M1 A1 E1 [5]	 for either A or B/C for both A and B/C
(ii) $\frac{dy}{d\theta} = -4\sin\theta$ $\frac{dx}{d\theta} = 2 - \cos\theta$ ⇒ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= -\frac{4\sin\theta}{2-\cos\theta}$ At A, gradient $= -\frac{4\sin(\pi/2)}{2-\cos(\pi/2)} = -2$	B1 M1 A1 A1 [4]	either $dx/d\theta$ or $dy/d\theta$ www
(iii) $\frac{dy}{dx} = 1 \Rightarrow -\frac{4\sin\theta}{2-\cos\theta} = 1$ ⇒ $-4\sin\theta = 2 - \cos\theta$ ⇒ $\cos\theta - 4\sin\theta = 2$ *	M1 E1 [2]	their $dy/dx = 1$
(iv) $\cos\theta - 4\sin\theta = R\cos(\theta + \alpha)$ $= R(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$ ⇒ $R\cos\alpha = 1$, $R\sin\alpha = 4$ ⇒ $R^2 = 1^2 + 4^2 = 17$, $R = \sqrt{17}$ $\tan\alpha = 4$, $\alpha = 1.326$ ⇒ $\sqrt{17}\cos(\theta + 1.326) = 2$ ⇒ $\cos(\theta + 1.326) = 2/\sqrt{17}$ ⇒ $\theta + 1.326 = 1.064, 5.219, 7.348$ ⇒ $\theta = (-0.262), 3.89, 6.02$	 M1 B1 M1 A1 M1 A1 A1 [7]	 corr pairs accept 76.0º , 1.33 radians inv cos $(2/\sqrt{17})$ ft their R for method -1 extra solutions in the range

PhysicsAndMathsTutor.com

6 $\frac{dy}{dt} = -a(1+t^2)^{-2}.2t$ $\frac{dx}{dt} = 3at^2$ $\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2at}{3at^2(1+t^2)^2}$ $= \frac{-2}{3t(1+t^2)^2}$ * At $(a, ½ a)$, $t = 1$ $\Rightarrow$ gradient = $\frac{-2}{3\times 2^2}$ = −1/6	M1 A1 B1 M1 E1 M1 A1 [7]	$(1+t^2)^{-2} \times kt$ for method ft finding t

7(i) dx/du = 2u , dy/du = 6u^2 $\Rightarrow \frac{dy}{dx} = \frac{dy/du}{dx/du} = \frac{6u^2}{2u}$ = 3u **OR** $y=2(x\text{-}1)^{3/2}$, $dy/dx=3(x\text{-}1)^{1/2}=3u$	B1 M1 A1 [3]	both 2u and 6u^2 B1(y=f(x)), M1 differentiation, A1
(ii) At (5, 16), u = 2 $\Rightarrow$ dy/dx = 6	M1 A1 [2]	cao

8 $x = \frac{1}{t} - 1 \Rightarrow \frac{1}{t} = x + 1$ $\Rightarrow t = \frac{1}{x+1}$ $\Rightarrow y = \frac{2 + \frac{1}{x+1}}{1 + \frac{1}{x+1}} = \frac{2x+2+1}{x+1+1} = \frac{2x+3}{x+2}$	M1 A1 M1 E1	Solving for t in terms of x or y Subst their t which must include a fraction, clearing subsidiary fractions/ changing the subject oe www
or $\frac{3+2x}{2+x} = \frac{3 + \frac{2-2t}{t}}{2 + \frac{1-t}{t}}$ $= \frac{3t+2-2t}{2t+1-t}$ $= \frac{t+2}{t+1} = y$	M1 A1 M1 E1 [4]	substituting for x or y in terms of t clearing subsidiary fractions/changing the subject

PhysicsAndMathsTutor.com