1 Fig. 8 shows a cross-section of a car headlight whose inside reflective surface is modelled, in suitable units, by the curve

$$x = 2t^2,\ y = 4t, \quad -\sqrt{2} \leqslant t \leqslant \sqrt{2}\ .$$

$P(2t^2, 4t)$ is a point on the curve with parameter t. TS is the tangent to the curve at P, and PR is the line through P parallel to the x-axis. Q is the point (2, 0). The angles that PS and QP make with the positive x-direction are θ and ϕ respectively.

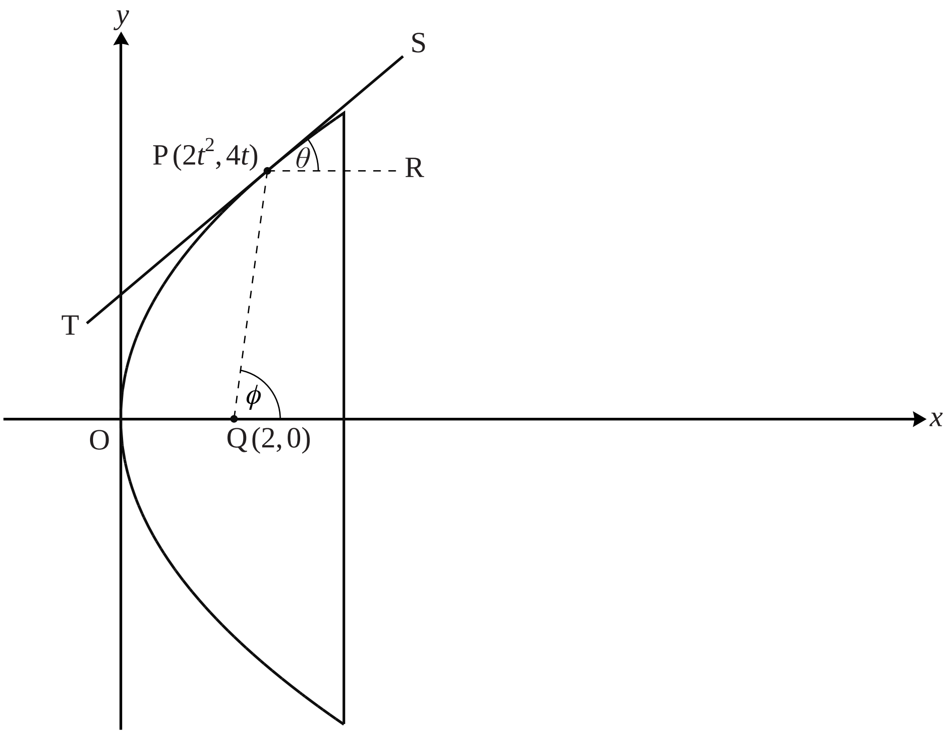

Fig. 8

(i) By considering the gradient of the tangent TS, show that $\tan\theta = \frac{1}{t}$. **[3]**

(ii) Find the gradient of the line QP in terms of t. Hence show that $\phi = 2\theta$, and that angle TPQ is equal to θ. **[8]**

[The above result shows that if a lamp bulb is placed at Q, then the light from the bulb is reflected to produce a parallel beam of light.]

The inside surface of the headlight has the shape produced by rotating the curve about the x-axis.

(iii) Show that the curve has cartesian equation $y^2 = 8x$. Hence find the volume of revolution of the curve, giving your answer as a multiple of π. **[7]**

2 Fig. 3 shows part of the curve $y = 1 + x^2$, together with the line $y = 2$.

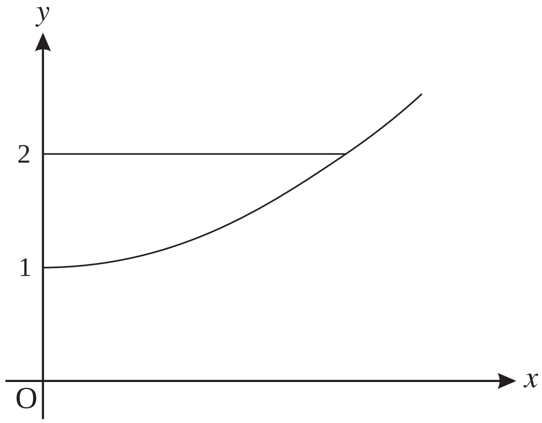

Fig. 3

The region enclosed by the curve, the y-axis and the line $y = 2$ is rotated through $360°$ about the y-axis. Find the volume of the solid generated, giving your answer in terms of π. [5]

3 Fig. 7 shows the curve BC defined by the parametric equations

$$x = 5\ln u,\ y = u + \frac{1}{u}, \quad 1 \leqslant u \leqslant 10.$$

The point A lies on the x-axis and AC is parallel to the y-axis. The tangent to the curve at C makes an angle θ with AC, as shown.

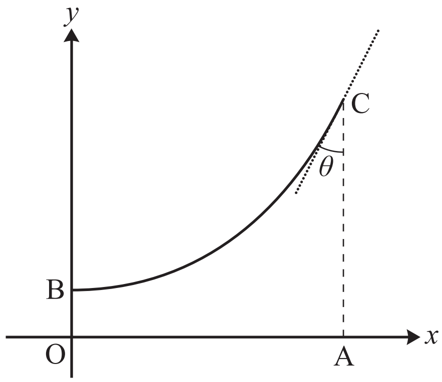

Fig. 7

(i) Find the lengths OA, OB and AC. **[5]**

(ii) Find $\frac{dy}{dx}$ in terms of u. Hence find the angle θ. **[6]**

(iii) Show that the cartesian equation of the curve is $y = e^{\frac{1}{5}x} + e^{-\frac{1}{5}x}$. **[2]**

An object is formed by rotating the region OACB through 360° about Ox.

(iv) Find the volume of the object. **[5]**

4 Fig. 2 shows the curve $y = \sqrt{1 + x^2}$.

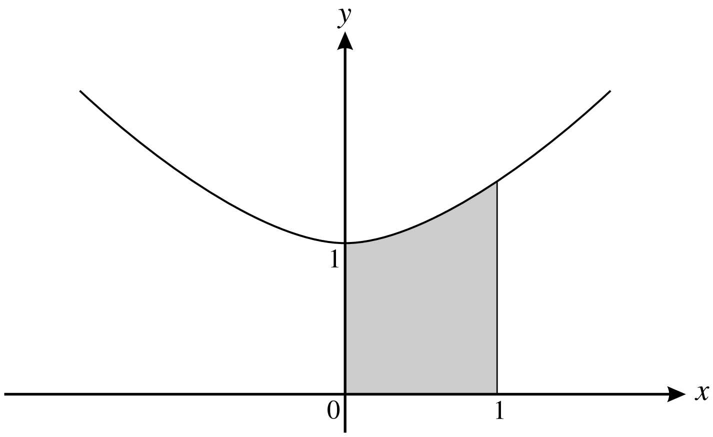

Fig. 2

(i) The following table gives some values of x and y.

x	0	0.25	0.5	0.75	1
y	1	1.0308		1.25	1.4142

Find the missing value of y, giving your answer correct to 4 decimal places.

Hence show that, using the trapezium rule with four strips, the shaded area is approximately 1.151 square units. **[3]**

(ii) Jenny uses a trapezium rule with 8 strips, and obtains a value of 1.158 square units. Explain why she must have made a mistake. **[2]**

(iii) The shaded area is rotated through 360° about the x-axis. Find the exact volume of the solid of revolution formed. **[3]**

5 Fig. 4 shows the curve $y = \sqrt{1 + e^{2x}}$, and the region between the curve, the x-axis, the y-axis and the line $x = 2$.

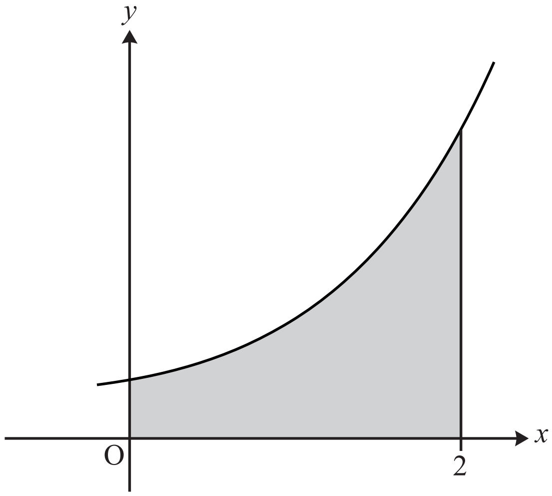

Fig. 4

(a) Find the exact volume of revolution when the shaded region is rotated through 360° about the x-axis. **[4]**

(b) **(i)** Complete the table of values, and use the trapezium rule with 4 strips to estimate the area of the shaded region. **[3]**

x	0	0.5	1	1.5	2
y		1.9283	2.8964	4.5919	

(ii) The trapezium rule for $\int_0^2 \sqrt{1 + e^{2x}}\,dx$ with 8 and 16 strips gives 6.797 and 6.823, although not necessarily in that order. Without doing the calculations, say which result is which, explaining your reasoning. **[1]**

6 Fig. 8 illustrates a hot air balloon on its side. The balloon is modelled by the volume of revolution about the x-axis of the curve with parametric equations

$$x = 2 + 2\sin\theta, \quad y = 2\cos\theta + \sin 2\theta, \quad (0 \leqslant \theta \leqslant 2\pi).$$

The curve crosses the x-axis at the point A (4, 0). B and C are maximum and minimum points on the curve. Units on the axes are metres.

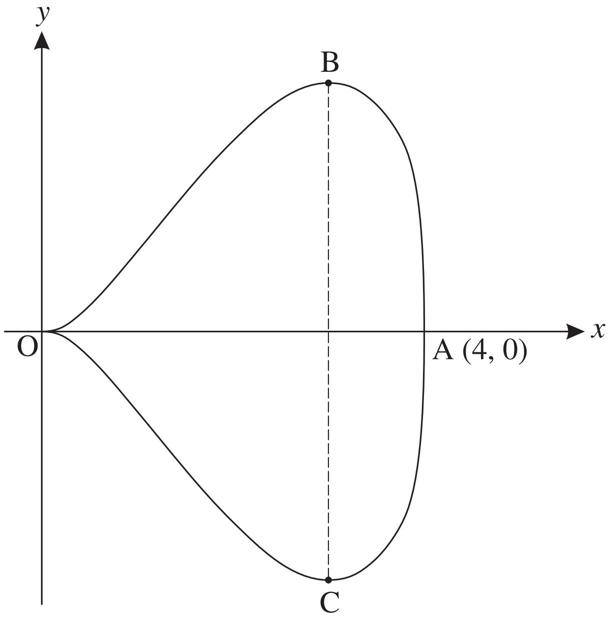

Fig. 8

(i) Find $\frac{dy}{dx}$ in terms of θ. **[4]**

(ii) Verify that $\frac{dy}{dx} = 0$ when $\theta = \frac{1}{6}\pi$, and find the exact coordinates of B.

Hence find the maximum width BC of the balloon. **[5]**

(iii) (A) Show that $y = x\cos\theta$.

(B) Find $\sin\theta$ in terms of x and show that $\cos^2\theta = x - \frac{1}{4}x^2$.

(C) Hence show that the cartesian equation of the curve is $y^2 = x^3 - \frac{1}{4}x^4$. **[7]**

(iv) Find the volume of the balloon. **[3]**