1	**(i)**		$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}=\frac{4}{4t}=\frac{1}{t}$ But gradient of tangent = tan θ * $\Rightarrow$ tan θ = 1/t	M1 A1 A1 **[3]**	their dy/dt /dx/dt accept 4/4t here **ag** -need reference to gradient is tan θ

Question			**answer**	**Marks**	**Guidance**
1	**(ii)**		Gradient of QP = $\frac{4t}{2t^2-2}=\frac{2t}{t^2-1}$	M1 A1	correct method for subtracting co-ordinates correct (does not need to be cancelled)
			$=\frac{2\frac{1}{\tan\theta}}{\frac{1}{\tan^2\theta}-1}$	M1	eithe substituting t=1/tanθ in above expression or substituting tanθ=1/t in double angle formula for tan2θ. (tan2θ=2tanθ/(1-tan²θ)=2/t/(1-1/t²)=2t/(t²-1)
			$=\frac{2\tan\theta}{1-\tan^2\theta}=\tan 2\theta$	A1	showing expressions are equal
			$\Rightarrow$ tan ϕ = tan 2θ $\Rightarrow$ ϕ = 2θ *	 A1	 **ag**
			$\Rightarrow$ Angle QPR = 180 – 2θ	M1	supplementary angles oe
			$\Rightarrow$ ∠TPQ + 180 – 2θ + θ = 180	M1	angles on a straight line oe
			$\Rightarrow$ ∠TPQ = θ *	A1 **[8]**	**ag**
1	**(iii)**		$t = y/4$ $\Rightarrow$ $x = 2\,y^2/16 = y^2/8$ $\Rightarrow$ $y^2 = 8x$*	 M1 A1	 e minating t from parametric equation **ag**
			When $t = \sqrt{2}$, $x = 2\times(\sqrt{2})^2 = 4$	B1	
			So $V = \int_0^4 \pi y^2\,\mathrm{d}x = \int_0^4 8\pi x\,\mathrm{d}x$	M1 A1	for M1 allow no limits or their limits need correct limits but they may appear later
			$=\left[4\pi x^2\right]_0^4$	B1	fo $4\pi x^2$ (ignore incorrect or missing limits)
			$= 64\pi$	A1 **[7]**	in terms of π only allow SC B1 for omission of π throughout integral but otherwise correct

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2 $V = \int_1^2 \pi x^2 dy$ $y = 1 + x^2 \Rightarrow x^2 = y - 1$ $\Rightarrow V = \int_1^2 \pi (y-1) dy$ $= \pi \left[\frac{1}{2}y^2 - y\right]_1^2$ $= \pi(2 - 2 - ½ + 1)$ $= ½ \pi$	B1 M1 B1 M1 A1 [5]	 $\left[\frac{1}{2}y^2 - y\right]$ substituting limits into integrand

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3	**(i)**		u = 10, x = 5 ln 10 = 11.5	M1	Using $u = 10$ to find OA
			so OA = $5\ln 10$	A1	accept 11.5 or better
			when u = 1,	M1	Using $u = 1$ to find OB or $u = 10$ to find AC
			y = 1 + 1 = 2 so OB = 2	A1	
			When u = 10, y = 10 + 1/10 = 10.1		
			So AC = 10.1	A1	
					In the case where values are given in coordinates instead of OA=,OB=,AC=, then give A0 on the first occasion this happens but allow subsequent As. Where coordinates are followed by length eg B(0, 2), length=2 then allow A1.
				[5]	

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Question			Answer	Marks	Guidance
3	(ii)		$\frac{dy}{dx} = \frac{dy/du}{dx/du} = \frac{1 - 1/u^2}{5/u}$ $\left[= \frac{u^2 - 1}{5u} \right]$	M1 A1	their dy/du /dx/du Award A1 if **any** correct form is seen at any stage including unsimplified (can isw)
			EITHER When u = 10, dy/dx = 99/50 = 1.98 tan (90 – θ)= 1.98 ⇒ θ = 90 – 63.2 = 26.8°	 M1 M1 A2	 **substituting u =10 in their expression** **or by geometry**, say using a triangle and the gradient of the line 26.8º, or 0.468 radians (or better) cao **SC** M1M0A1A0 for 63.2º (or better) or 1.103 radians(or better)
			OR When u = 10, dy/dx = 99/50 = 1.98 tan(90– θ) = 99/50 ⇒ tanθ = 50/99 θ = 26.8º	 M1 M1 A2 **[6]**	 **allow use of their expression for M marks** 26.8º, or 0.468 radians (or better) cao
3	(iii)		$x = 5 \ln u \Rightarrow x/5 = \ln u$, $u = e^{x/5}$ $\Rightarrow \quad y = u + 1/u = e^{x/5} + e^{-x/5}$	M1 A1 **[2]**	Need some working Need some working as AG

Question			Answer	Marks	Guidance
3	**(iv)**		Vol of rev $= \int_0^{5\ln 10} \pi y^2 \mathrm{d}x = \int_0^{5\ln 10} \pi (e^{x/5} + e^{-x/5})^2 \mathrm{d}x$	M1	need π $(e^{x/5}+e^{-x/5})^2$ and dx soi. Condone wrong limits or omission of limits for M1. Allow M1 if y prematurely squared as eg $(e^{2x/5}+e^{-2x/5})$
			$= \int_0^{5\ln 10} \pi (\mathrm{e}^{2x/5} + 2 + \mathrm{e}^{-2x/5}) \mathrm{d}x$	A1	including **correct** limits at some stage (condone 11.5 for this mark)
			$= \pi \left[(\frac{5}{2}\mathrm{e}^{2x/5} + 2x - \frac{5}{2}\mathrm{e}^{-2x/5}) \right]_0^{5\ln 10}$	B1	$[\frac{5}{2}e^{2x/5}+2x-\frac{5}{2}e^{-2x/5}]$ allow if no π and/or no limits or incorrect limits
			$= \pi(250 + 10\ln 10 - 0.025 - 0)$	M1	substituting both limits (their OA and 0) in an expression of correct form ie $ae^{2x/5}+be^{-2x/5}+cx$, $a,b,c \neq 0$ and subtracting in correct order (- 0 is sufficient for lower limit) Condone absence of π for M1
			$= 858$	A1	accept 273π and answers rounding to 273π or 858
				[5]	NB The integral can be evaluated using a change of variable to u. This involves changing dx to (dx/du)x du. For completely correct work from this method award full marks. Partially correct solutions must include the change in dx. If in doubt consult your TL. **Remember to indicate second box has been seen even if it has not been used.**

4(i) When $x = 0.5$**,** $y = 1.1180$ $\Rightarrow$ $A \approx 0.25/2\{1+1.4142+2(1.0308+1.1180+1.25)\}$ $= 0.25 \times 4.6059 = 1.151475$ $= 1.151$ (3 d.p.)*	B1 M1 E1 [3]	4dp (0.125 x 9.2118) need evidence
(ii) Explain that the area is an over-estimate. *or* The curve is below the trapezia, so the area is an over- estimate. This becomes less with more strips. *or* Greater number of strips improves accuracy so becomes less	B1 B1 [2]	or use a diagram to show why
(iii) $V = \int_0^1 \pi y^2 dx$ $= \int_0^1 \pi(1+x^2)dx$ $= \pi\left[(x+x^3/3)\right]_0^1$ $= 1\frac{1}{3}\pi$	 M1 B1 A1 [3]	 allow limits later $x + x^3/3$ exact

Question			Answer	Marks	Guidance
5	**(a)**		$V = \int_0^2 \pi y^2 \mathrm{d}x = \int_0^2 \pi(1+\mathrm{e}^{2x})\mathrm{d}x$	M1	$\int_0^2 \pi(1+\mathrm{e}^{2x})\mathrm{d}x$ limits must appear but may be later condone omission of *dx* if intention clear
			$= \pi\left[x + \frac{1}{2}\mathrm{e}^{2x}\right]_0^2$	B1	$\left[x + \frac{1}{2}\mathrm{e}^{2x}\right]$ independent of π and limits
			$= \pi(2 + ½\ \mathrm{e}^4 – ½\ )$	DM1	dependent on first M1.Need **both** limits substituted in their integral of the form $ax + b\,e^{2x}$, where a, b non-zero constants. Accept answers including e^0 for M1. Condone absence of π for M1 at this stage
			$= ½\ \pi(3 + \mathrm{e}^4)$	A1 **[4]**	cao exact only
5	**(b)**	**(i)**	$x = 0$, $y = 1.4142$; $x = 2$, $y = 7.4564$	B1	1.414, 7.456 or better
			$A = 0.5/2\{(1.4142 + 7.4564)$ $+ 2(1.9283 + 2.8964 + 4.5919)\}$	M1	correct formula seen (can be implied by correct intermediate step eg 27.7038../4)
			$= 6.926$	A1 **[3]**	6.926 or 6.93 (do not allow more dp)
5	**(b)**	**(ii)**	8 strips: 6.823, 16 strips: 6.797 Trapezium rule overestimates this area, but the overestimate gets less as the no of strips increases.	B1 **[1]**	oe

6(i) $\frac{dy}{d\theta} = 2\cos 2\theta - 2\sin\theta$, $\frac{dx}{d\theta} = 2\cos\theta$	B1, B1	
$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$	M1	substituting for theirs
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\cos 2\theta - 2\sin\theta}{2\cos\theta} = \frac{\cos 2\theta - \sin\theta}{\cos\theta}$	A1 [4]	oe
(ii) When $\theta = \pi/6$, $\frac{dy}{dx} = \frac{\cos\pi/3 - \sin\pi/6}{\cos\pi/6}$ $= \frac{1/2 - 1/2}{\sqrt{3}/2} = 0$	E1	
Coords of B: $x = 2 + 2\sin(\pi/6) = 3$ $y = 2\cos(\pi/6) + \sin(\pi/3) = 3\sqrt{3}/2$	M1 A1,A1	for either exact
BC = $2 \times 3\sqrt{3}/2 = 3\sqrt{3}$	B1ft [5]	
(iii) (*A*) $y = 2\cos\theta + \sin 2\theta$ $= 2\cos\theta + 2\sin\theta\cos\theta$	M1	$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\cos\theta(1 + \sin\theta)$ $= x\cos\theta$ *	E1	
(*B*) si $\theta = ½(x - 2)$	B1	
$\cos^2\theta = 1 - \sin^2\theta$ $= 1 - ¼(x-2)^2$ $= 1 - ¼x^2 + x - 1$	M1	
$= (x - ¼x^2)$ *	E1	
(*C*) artesian equation is $y^2 = x^2\cos^2\theta$	M1	squaring and substituting for x
$= x^2(x - ¼x^2)$ $= x^3 - ¼x^4$*	E1 [7]	
(iv) $V = \int_0^4 \pi y^2 dx$ $= \pi\int_0^4 (x^3 - \frac{1}{4}x^4)dx$	M1	need limits
$= \pi\left[\frac{1}{4}x^4 - \frac{1}{20}x^5\right]_0^4$	B1	$\left[\frac{1}{4}x^4 - \frac{1}{20}x^5\right]$
$= \pi(64 - 51.2)$ $= 12.8\pi = 40.2$ (m^3)	A1 [3]	12.8π or 40 or better.