1 Fig. 6 shows the region enclosed by the curve $y = (1 + 2x^2)^{\frac{1}{3}}$ and the line $y = 2$.

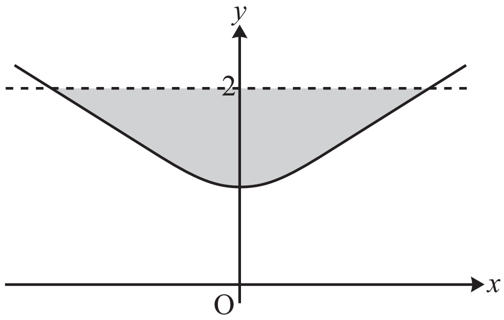

Fig. 6

This region is rotated about the y-axis. Find the volume of revolution formed, giving your answer as a multiple of π. **[6]**

2 Fig. 7a shows the curve with the parametric equations

$$x = 2\cos\theta, \quad y = \sin 2\theta, \quad -\tfrac{\pi}{2} \leqslant \theta \leqslant \tfrac{\pi}{2}.$$

The curve meets the x-axis at O and P. Q and R are turning points on the curve. The scales on the axes are the same.

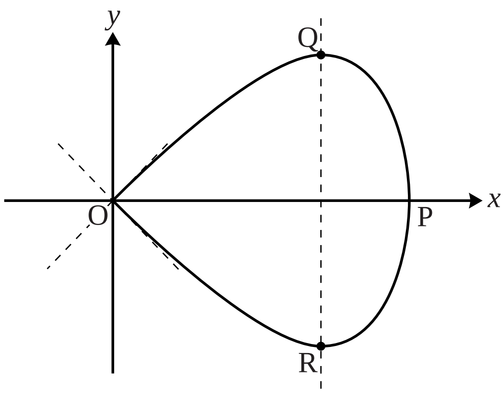

Fig. 7a

(i) State, with their coordinates, the points on the curve for which $\theta = -\frac{\pi}{2}$, $\theta = 0$ and $\theta = \frac{\pi}{2}$. **[3]**

(ii) Find $\dfrac{dy}{dx}$ in terms of θ. Hence find the gradient of the curve when $\theta = \frac{\pi}{2}$, and verify that the two tangents to the curve at the origin meet at right angles. **[5]**

(iii) Find the exact coordinates of the turning point Q. **[3]**

When the curve is rotated about the x-axis, it forms a paperweight shape, as shown in Fig. 7b.

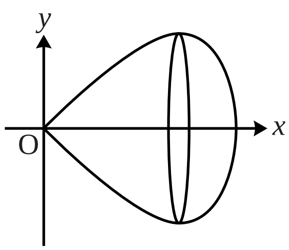

Fig. 7b

(iv) Express $\sin^2\theta$ in terms of x. Hence show that the cartesian equation of the curve is $y^2 = x^2(1 - \frac{1}{4}x^2)$. **[4]**

(v) Find the volume of the paperweight shape. **[4]**

3 Fig. 6 shows the region enclosed by part of the curve $y = 2x^2$, the straight line $x + y = 3$, and the y-axis. The curve and the straight line meet at P (1, 2).

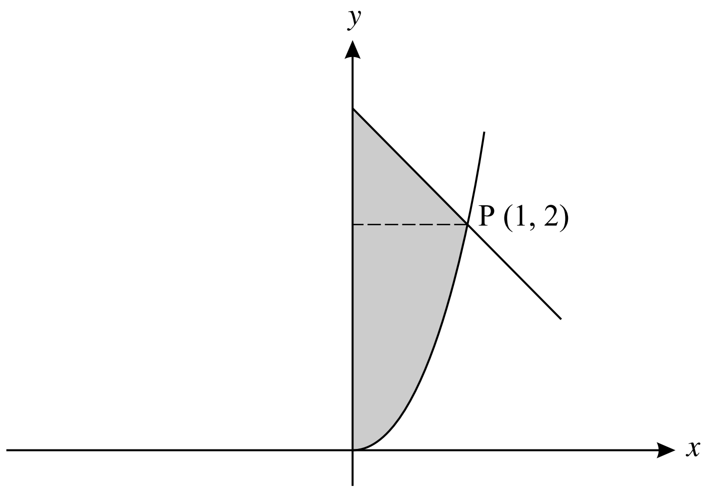

Fig. 6

The shaded region is rotated through 360° about the y-axis. Find, in terms of π, the volume of the solid of revolution formed. **[7]**

[You may use the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.]

4 The part of the curve $y = 4 - x^2$ that is above the x-axis is rotated about the y-axis. This is shown in Fig. 4.

Find the volume of revolution produced, giving your answer in terms of π. **[5]**

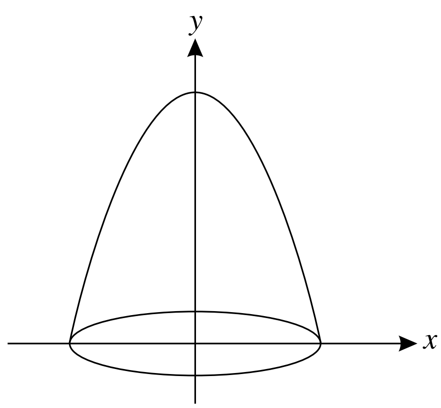

Fig. 4

5 Fig. 2 shows the curve $y = \sqrt{1 + e^{2x}}$.

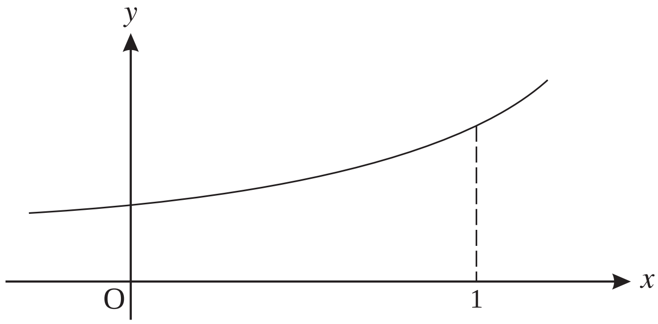

Fig. 2

The region bounded by the curve, the x-axis, the y-axis and the line $x = 1$ is rotated through 360° about the x-axis.

Show that the volume of the solid of revolution produced is $\frac{1}{2}\pi(1 + e^2)$. [4]

6 Fig. 3 shows the curve $y = \ln x$ and part of the line $y = 2$.

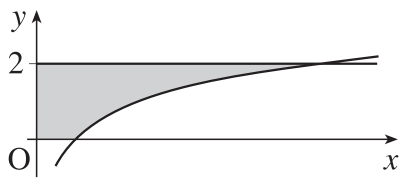

Fig. 3

The shaded region is rotated through 360° about the y-axis.

(i) Show that the volume of the solid of revolution formed is given by $\int_0^2 \pi e^{2y}\,dy$. [3]

(ii) Evaluate this, leaving your answer in an exact form. [3]

7 **(i)** Show that $\int x\mathrm{e}^{-2x}\,\mathrm{d}x = -\frac{1}{4}\mathrm{e}^{-2x}(1+2x) + c$. [3]

A vase is made in the shape of the volume of revolution of the curve $y = x^{\frac{1}{2}}\mathrm{e}^{-x}$ about the x-axis between $x = 0$ and $x = 2$ (see Fig. 5).

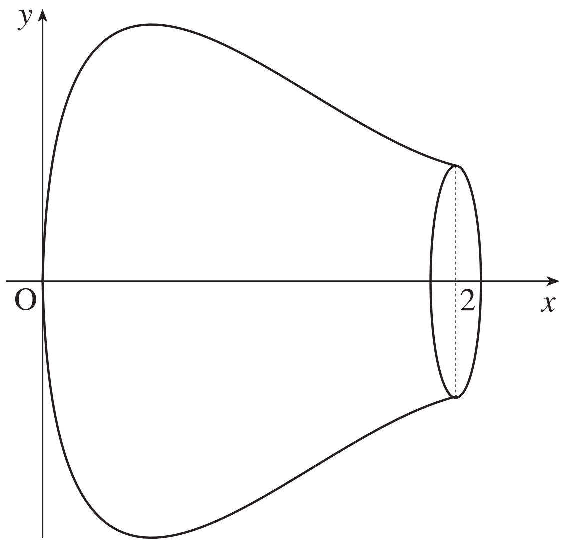

Fig. 5

(ii) Show that this volume of revolution is $\frac{1}{4}\pi\left(1 \quad \frac{5}{\mathrm{e}^4}\right)$. [4]

8 Fig. 4 shows a sketch of the region enclosed by the curve $\sqrt{1 + e^{-2x}}$, the x-axis, the y-axis and the line $x = 1$.

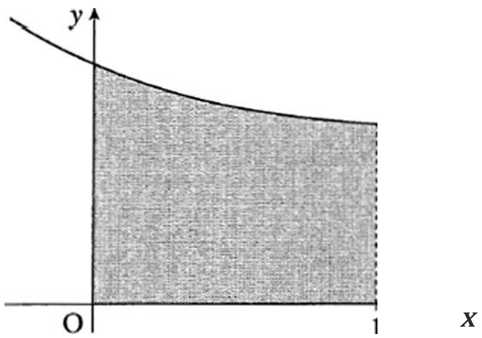

Fig. 4

Find the volume of the solid generated when this region is rotated through 360° about the ***x*-axis**. Give your answer in an exact form. (5)