Question			Answer	Marks	Guidance
1			$y=(1+2x^2)^{\frac{1}{3}} \Rightarrow y^3 = 1+2x^2$		
			$\Rightarrow x^2 = \frac{1}{2}(y^3-1)$	M1	finding x^2 (or x) correctly in terms of y
			$V = \int_1^2 \pi x^2 \, \mathrm{d}y = \frac{1}{2}\pi\int_1^2 (y^3-1)\,\mathrm{d}y$	M1	For M1 need $\int \pi x^2 \mathrm{d}y$ with substitution for their x^2 (in terms of y only) Condone absence of dy throughout if intentions clear. (need π)
				A1	www For A1 it must be correct with correct limits 1 and 2, but they may appear later
			$= \frac{1}{2}\pi\left[\frac{1}{4}y^4 - y\right]_1^2 = \frac{1}{2}\pi(2+\frac{3}{4})$	B1	$1/2[y^4/4 - y]$ independent of π and limits
				M1	substituting both their limits in correct order in correct expression, condone a minor slip for M1 (if using $y = 0$ as lower limit then '-0' is enough) condone absence of π for M1
			$= \frac{11}{8}\pi$	A1	oe exact only www (1⅜ π or 1.375π)
				[6]	

Question			Answer	Marks	Guidance
2	(i)		$\theta = -\pi/2$: O (0, 0) $\theta = 0$: P (2, 0) $\theta = \pi/2$: O (0, 0)	B1 B1 B1 **[3]**	Origin or O, condone omission of (0, 0) or O Or, say at P $x = 2$, $y = 0$, need P stated Origin or O, condone omission of (0,0) or O
2	(ii)		$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{2\cos 2\theta}{-2\sin\theta} = -\frac{\cos 2\theta}{\sin\theta}$ When $\theta = \pi/2$ $dy/dx = -\cos\pi/ \sin\pi/2 = 1$ When $\theta = -\pi/2$ $dy/dx = -\cos(-\pi)/ \sin(-\pi/2) = -1$ **Either** $1\times -1 = -1$ so perpendicular **Or** gradient tangent $=1 \Rightarrow$ meets axis at 45° , similarly, gradient $= -1 \Rightarrow$ meets axis at 45° oe	M1 A1 M1 A1 A1 **[5]**	their $dy/d\theta$ $/dx/d\theta$ any equivalent form www (not from $-2\cos 2\theta/2\sin\theta$) subst $\theta = \pi/2$ in their equation Obtaining $dy/dx = 1$,and $dy/dx = -1$ **shown** (or explaining using symmetry of curve) **www** justification that tangents are perpendicular **www** dependent on previous A1
2	(iii)		At Q, $\sin 2\theta = 1 \Rightarrow 2\theta = \pi/2$, $\theta = \pi/4$ $\Rightarrow$ coordinates of Q are $(2\cos\pi/4, \sin\pi/2)$ $= (\sqrt{2}, 1)$	M1 A1 A1 **[3]**	or, using the derivative, $\cos 2\theta = 0$ soi or their $dy/dx = 0$ to find θ. If the only error is in the sign or the coeff of the derivative in (ii), allow full marks in this part (condone $\theta = 45°$) www (exact only) accept $2/\sqrt{2}$
2	(iv)		$\sin^2\theta = (1 - \cos^2\theta) = 1 - ¼x^2$ $\Rightarrow$ $y = \sin 2\theta = 2\sin\theta\cos\theta$ $= (\pm) x\sqrt{(1 - ¼x^2)}$ $\Rightarrow$ $y^2 = x^2(1 - ¼x^2)$*	B1 M1 A1 A1 **[4]**	oe, eg may be $x^2 =$….. **Use** of $\sin 2\theta = 2\sin\theta\cos\theta$ subst for x **or** $y^2 = 4\sin^2\theta\cos^2\theta$ (squaring) either order oe squaring **or** subst for x either order oe **AG**

Question			Answer	Marks	Guidance
2	(v)		$V = \int_0^2 \pi x^2 (1 - \frac{1}{4}x^2)\,\mathrm{d}x$	M1	integral including correct limits but ft their '2' from (i) (limits may appear later) condone omission of dx if intention clear
			$= \int_0^2 (\pi x^2 - \frac{1}{4}\pi x^4)\,\mathrm{d}x$		
			$= \pi\left[\frac{1}{3}x^3 - \frac{1}{20}x^5\right]_0^2$	B1	$\left[\frac{1}{3}x^3 - \frac{1}{20}x^5\right]$ ie allow if no π and/or incorrect/no limits (or equivalent by parts)
			$= \pi\left[\frac{8}{3} - \frac{32}{20}\right]$	A1	substituting limits into correct expression (including π) ft their '2'
			$= 16\pi/15$	A1 **[4]**	cao oe, 3.35 or better (any multiple of π must round to 3.35 or better)

3	Vol = vol of rev of curve + vol of rev of line vol of rev of curve = $\int_0^2 \pi x^2 \,\mathrm{d}y$ $= \int_0^2 \pi \frac{y}{2} \,\mathrm{d}y$ $= \pi \left[\frac{y^2}{4} \right]_0^2$ $= \pi$ height of cone = 3 – 2 = 1 so vol of cone = 1/3 π 1^2 x1 = π/3 so total vol = 4π/3	M1 M1 B1 A1 B1 B1 A1 [7]	(soi) at any stage substituting $x^2 = y/2$ $\left[\frac{y^2}{4} \right]$ h=1 soi www cao	 for M1 need *π*, substitution for x^2, (*dy* soi), intention to integrate and correct limits even if *π* missing or limits incorrect or missing cao OR $\pi \int_2^3 (3-y)^2 dy$ M1(even if expanded incorrectly) =π/3 A1 www

4 When $x = 0, y = 4$	B1	
$\Rightarrow V = \pi\int_0^4 x^2 dy$	M1	must have integral, π, x^2 and dy soi
$= \pi\int_0^4 (4-y)dy$	M1	must have π ,their (4-y), their numerical y limits
$= \pi\left[4y - \frac{1}{2}y^2\right]_0^4$	B1	$\left[4y - \frac{1}{2}y^2\right]$
$= \pi(16 - 8) = 8\pi$	A1 [5]	

5 $V = \int_0^1 \pi y^2 dx = \int_0^1 \pi(1+e^{2x})dx$	M1	must be π x their y^2 in terms of x
$= \pi\left[x + \frac{1}{2}e^{2x}\right]_0^1$	B1	$\left[x + \frac{1}{2}e^{2x}\right]$ only
$= \pi(1 + \frac{1}{2}e^2 - \frac{1}{2})$	M1	substituting both x limits in a function of x
$= \frac{1}{2}\pi(1+e^2)$ *	E1 [4]	www

6 **(i)** $y = \ln x \Rightarrow x = e^y$	B1	
$\Rightarrow V = \int_0^2 \pi x^2 dy$	M1	
$= \int_0^2 \pi(e^y)^2 dy = \int_0^2 \pi e^{2y} dy$ *	E1 [3]	
(ii) $\int_0^2 \pi e^{2y} dy = \pi\left[\frac{1}{2}e^{2y}\right]_0^2$	B1	½ e^{2y}
	M1	substituting limits in $k\pi e^{2y}$
$= ½\,\pi(e^4 - 1)$	A1 [3]	or equivalent, but must be exact and evaluate e^0 as 1.

7 (i) $\int xe^{-2x}dx$ let $u = x$, $dv/dx = e^{-2x}$ $\Rightarrow v = -½ e^{-2x}$ $= -\frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}dx$ $= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + c$ $= -\frac{1}{4}e^{-2x}(1+2x)+c$ * or $\frac{d}{dx}[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + c] = -\frac{1}{2}e^{-2x} + xe^{-2x} + \frac{1}{2}e^{-2x}$ $= xe^{-2x}$	M1 A1 E1 M1 A1 E1 [3]	Integration by parts with $u = x$, $dv/dx = e^{-2x}$ $= -\frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}dx$ condone omission of c product rule
(ii) $V = \int_0^2 \pi y^2 dx$ $= \int_0^2 \pi (x^{1/2}e^{-x})^2 dx$ $= \pi \int_0^2 xe^{-2x}dx$ $= \pi\left[-\frac{1}{4}e^{-2x}(1+2x)\right]_0^2$ $= \pi(-¼ e^{-4}.5 + ¼)$ $= \frac{1}{4}\pi(1-\frac{5}{e^4})$ *	M1 A1 DM1 E1 [4]	Using formula condone omission of limits $y^2=xe^{-2x}$ condone omission of limits and π condone omission of π (need limits)

8 $V = \int \pi y^2 dx$ $= \int_0^1 \pi(1+e^{-2x})dx$ $= \pi\left[x - \frac{1}{2}e^{-2x}\right]_0^1$ $= \pi(1 - ½ e^{-2} + ½)$ $= \pi(1½ - ½ e^{-2})$	**M1** **M1** **B1** **M1** **A1** **[5]**	Correct formula $k\int_0^1 (1+e^{-2x})dx$ $\left[x - \frac{1}{2}e^{-2x}\right]$ substituting limits. Must see *0* used. Condone omission of π o.e. but must be exact